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Generalised Head-Discharge Equations for Culverts

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SUMMARY Generalised head-discharge equations are presented for culverts operating under inlet or outlet control. The equations are dimensionally correct and are based on the U.S. Department of Transportation methods.

1 INTRODUCTION

A review of culvert hydraulics by the author (Reference 2) indicated that the U.S. Department of Transportation design procedure (Reference 12) had the best basis in theory, and was supported by over 1000 experimental measurements of prototype and model culverts.

Results obtained using the U.S. Department of Transportation procedure were found to be consistent with other studies (References 1,3,4,7,10) and the method has been widely adopted in Australia and overseas (References 5,6,9,11).

The sequence of steps involved in the procedure is conveniently summarised in a design flow chart (Figure 1). The headwater level corresponding to a specified discharge may be read from a set of nomographs covering culvert discharges with inlet or outlet control, in box or circular pipe culverts. While these nomographs give quite adequate results, the increasing use of computers (and particularly personal computers) opens the opportunity to incorporate the design procedure into a computer based form. For this, generalised equations for both box and circular pipe culverts operating under inlet or outlet control are required. This paper describes the derivation of these equations, based on the experimental and theoretical results contained in the U.S. Department of Transportation procedure.

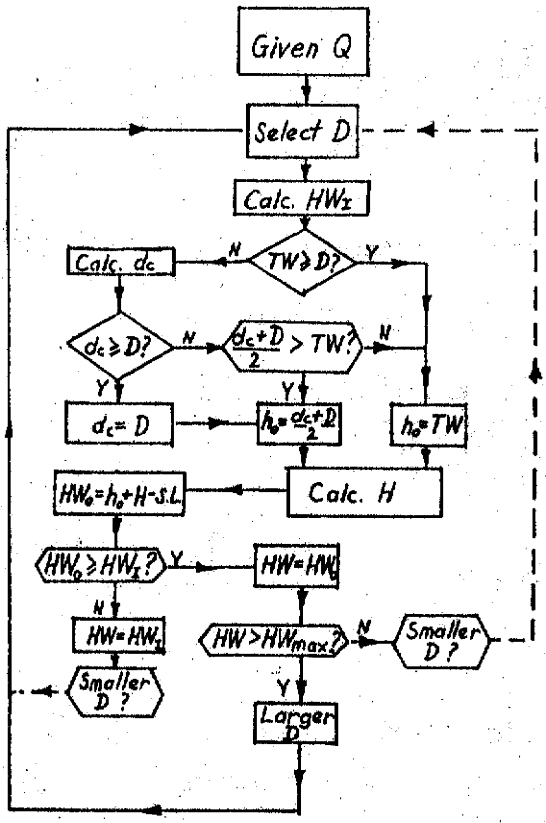


Figure 1 Culvert Design Flow Chart

S.I. units of metres and seconds are used throughout the text.

2 TYPES OF CULVERT FLOW

Figure 2 shows the range of flow types commonly encountered in culverts. The distinguishing features are inlet submergence by the headwater level HW and outlet submergence by the tailwater depth TW. The tailwater depth is determined by conditions in the downstream channel, including channel control sections, obstructions, uniform flow and water levels at a downstream confluence. Most natural channels are wide relative to the culvert and the tailwater depth is less than critical depth at the outlet, thus making the tailwater depth ineffective in the head-discharge relation.

The headwater level HW is set by the upstream energy line required to convey the flowrate through the culvert. Because the flow contracts on entering the culvert, the inlet will be submerged only when HW exceeds the culvert height D. This value of HW is generally in the range 1.2D to 1.5D, depending on the particular inlet geometry.

The most important factor in determining the head-discharge relation for a culvert is whether the flow is subject to inlet control or outlet control. The energy line in the headwater pond must provide sufficient energy to convey the flowrate into the culvert inlet, and also sufficient head difference above the tailwater to meet the entrance loss, friction loss in the barrel and exit loss. For a given flowrate, flow control is determined by the larger of the headwater depths required under inlet or outlet control.

Inlet control can occur with inlet submerged and outlet not submerged (Figure 2a). The flow contr-

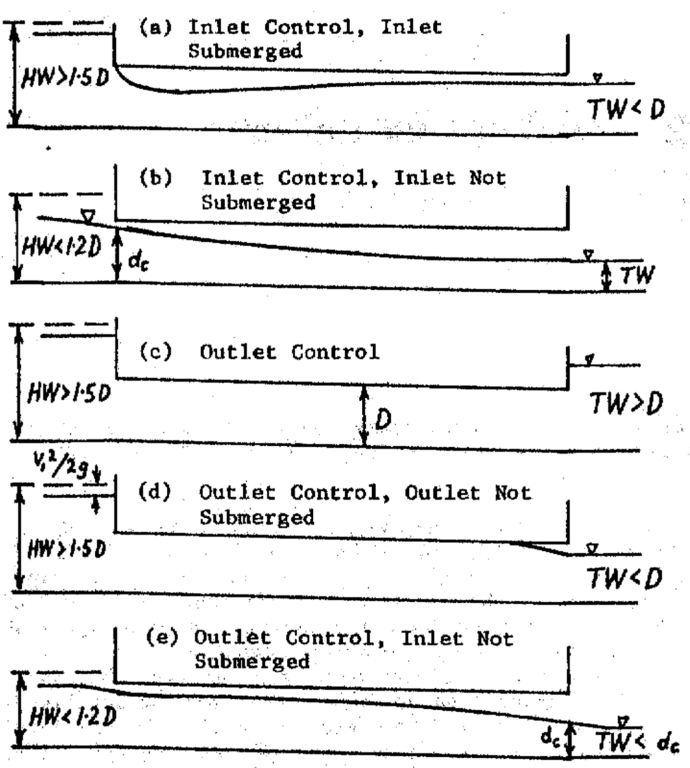


Figure 2 Types of Culvert Flow

acts to a supercritical jet immediately downstream of the inlet. If the culvert is laid at a steep slope, flow remains supercritical in the barrel. If the tailwater depth exceeds critical depth, a hydraulic jump can form near the outlet. Inlet control with both inlet and outlet not submerged (Figure 2b) passes through critical depth at the inlet, with supercritical flow in the barrel.

With outlet control and both inlet and outlet submerged (Figure 2c) the culvert flows full under pressure. The culvert can also flow full over part of its length, then part full at the outlet (Figure 2d). The point at which the water surface breaks away from the culvert crown depends on the tailwater depth and culvert grade, and can be determined using backwater calculation. If the culvert is laid at a flat grade, outlet control can occur with both inlet and outlet not submerged, and part full flow throughout the barrel (Figure 2e). In this case critical depth occurs at the outlet and the flow in the barrel is subcritical. Minor variations of these main types can occur, depending on the relative values of critical depth, normal depth, culvert diameter and tailwater depth.

A further variation is for the culvert to flow part full initially as in Figure 2a with the flow depth increasing until it touches the crown, after which full pipe flow occurs. This may occur in long culverts laid at flat grades. Varied flow calculations can be used to calculate the culvert length and grade at which this occurs, producing curves defining hydraulically long or short culverts, as given in Carter (1957) and reproduced in Chow (1959). These references also contain a full classification of flow types.

3 CRITICAL DEPTH IN BOX AND CIRCULAR PIPE CULVERTS

From the previous Section, critical depth can occur at the culvert inlet or outlet, and its value may be required for culverts discharging under inlet control or outlet control.

Critical depth occurs when the Froude number equals 1.0. For a box culvert of width B and height D, with discharge Q, the critical depth is

$$d_c = (Q^2/gB^2)^{0.333} = 0.4672\ (Q/B)^{0.667} \qquad (1)$$

For a circular pipe culvert, critical depth occurs when:

$$F = Q.B_c^{0.5}/g.^{0.5}A_c^{1.5} = 1 \qquad (2)$$

where B_c = surface width and A_c = flow cross section area at critical depth, as illustrated in Figure 3a.

Values of B_c and A_c depend on the critical depth d_c as follows

$$d_c = D(1 - \cos\theta)/2 \qquad (3a)$$

$$B_c = D.\sin\theta \qquad (3b)$$

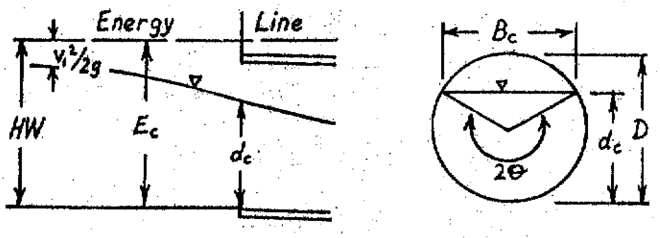
$$A_c = D^2(\theta - \frac{\sin 2\theta}{2})/4 \qquad (3c)$$

Equations (3a, b and c) can be substituted into (1), however the resulting equation is implicit and iteration is required to solve for d_c. A simpler, but approximate relation for critical depth in a circular pipe is

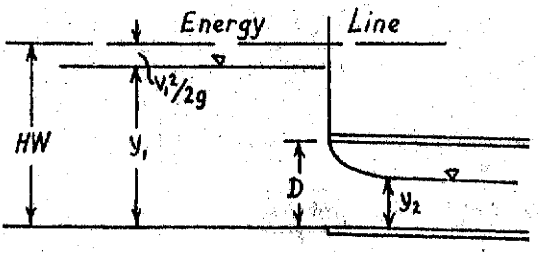
$$Q/g^{0.5}.D^{2.5} = 1.26.(d_c/D)^{3.75} \qquad 0.85 < \frac{d_c}{D} < 1 \qquad (4a)$$

$$Q/g^{0.5}.D^{2.5} = 0.95\ (d_c/D)^{1.95} \qquad 0 < \frac{d_c}{D} < 0.85 \qquad (4b)$$

Equation (4) has negligible error in dc for dc/D < 0.98 and 2% error for dc/D > 0.98.



(a) Inlet Not Submerged



(b) Inlet Submerged

Figure 3 Culvert with Inlet Control

4 DISCHARGE WITH INLET CONTROL

4.1 Inlet Control, Inlet Not Submerged

With culverts subject to inlet control, the important factors are the entrance conditions, including the entrance type, existence and angle of headwalls, projection of the culvert into the headwater pond, and the use of hood type inlets.

The head-discharge relation has two distinct regimes, inlet submerged and not submerged. With the inlet not submerged, the flow passes through critical depth at the inlet (Figure 3a) and the discharge can be calculated from this fact.

At the inlet, the location of the energy line above the culvert invert is given by

$$E_c = d_c + V_c^2/2g \qquad (5)$$

If energy losses between the headwater pond and the inlet are negligible, then E_c will be equal to the energy line in the headwater pond, denoted by HW. Note that HW refers to the energy line in the headwater pond. If, as is usually the case, the approach velocity head $v_1^2/2g$ is negligible, then HW corresponds the headwater depth. However if the approach velocity is significant then the headwater depth will be $V_1^2/2g$ less than HW. This definition of HW applies to all cases of culvert discharges considered in this paper.

From the above discussion, the critical depth corresponding to any given discharge Q can be calculated from equations (1) or (4) as appropriate, and the corresponding headwater level HW calculated from equation (5).

For a box culvert with inlet control, inlet not submerged the resulting head-discharge relation is

$$HW = 1.5.d_c = 1.5(Q^2/g\ B^2)^{0.333}$$

or

$$Q = 0.5443\ g^{0.5}.B.HW^{1.5} \qquad (6)$$

For a circular pipe culvert with inlet control, inlet not submerged, values of V_c must be calculated for the flow cross section area A_c corresponding to the particular value of d_c. Generalised equations for this case will be presented in Section 4.3.

4.2 Inlet Control, Inlet Submerged

In this case the inlet acts as a flow constriction and the head-discharge relation can be established by considering the energy equation, assuming no energy losses at the inlet.

In a box culvert, the inlet acts as a sluice gate (Henderson, 1966). Applying the Bernoulli equation between upstream ponded water and the vena contracta formed immediately downstream of the culvert entrance (Figure 3b) gives

$$HW = y_2 + V_2^2/2g = y_2 + Q^2/2g\ B^2 y_2^2 \qquad (7a)$$

The flow depth at the vena contracta is $y_2 = C_c.D$, where C_c is a contraction coefficient which is close to 0.6 for a square edge entrance. Re-arranging (7a) gives

$$Q = C_c.B.D\ [2g(HW - C_c.D)]^{0.5} \qquad (7b)$$

As for Section 4.1, the headwater level HW refers to the energy line in the headwater pond, and will be equal to the headwater depth if the approach velocity head $V_1^2/2g$ is negligible.

For a circular pipe culvert, the inlet acts an an orifice with a head-discharge relation

$$Q = C.\ \frac{\pi D^2}{4} \left[2g\ (HW - C_c.D) \right]^{0.5} \qquad (8)$$

4.3 Generalised Equations for Culverts with Inlet Control

The U.S. Department of Transportation nomographs contain equations 5 to 8, adjusted to fit experimental data, and incorporating three different types of entrance.

Generalised equation for culverts discharging under inlet control were derived by plotting points from these nomographs on logarithmic paper and fitting equations to the data. The resulting equations are:

Box Culvert, Type 1 Entrance

Inlet Not Submerged (HW/D<1.35)

$$Q=0.544g^{0.5}.B.HW_1^{1.50} \qquad (9a)$$

Inlet Submerged (HW/D>1.35)

$$Q=0.702g^{0.5}.B.D^{0.89}.HW_1^{0.61} \qquad (9b)$$

The relation between headwater levels HW_1, HW_2 and HW_3 for entrance types 1, 2 and 3 respectively are

$$HW_{2/D} = 1.09\ (HW_1/D)^{0.99} \qquad (9c)$$

$$HW_{3/D} = 1.07\ (HW_1/D)^{1.08} \qquad (9d)$$

where Entrance type 1 is Wingwall Flare 30° to 75°
Entrance type 2 is Wingwall Flare 90° and 15°
and Entrance type 3 is Wingwall Flare 0°

Equations 9 are used as follows: Given a discharge Q, calculate HW_1 from 9a or 9b, then calculate HW_2 or HW_3 from 9c or 9d as appropriate. Conversely, given HW_2, or HW_3 calculate HW_1 from 9c and 9d, then calculate Q from 9a or 9b.

Circular Pipe Culvert, Type 1 Entrance

Inlet Not Submerged (HW/D<1.2)

$$Q=0.421g^{0.5}.D^{0.87}.HW_1^{1.63} \qquad (10a)$$

Inlet Submerged (HW/D>1.2)

$$Q=0.530g^{0.5}D^{1.87}HW_1^{0.63} \qquad (10b)$$

The relation between headwater levels for Entrance types 1,2 and 3 are

$$HW_{2/D} = 0.92\ (HW_1/D)^{0.90} \qquad (10c)$$

$$HW_{3/D} = 0.91\ (HW_1/D)^{0.94} \qquad (10d)$$

where Entrance Type 1 is Square edge with headwall
Entrance Type 2 is Groove end with headwall
and Entrance Type 3 is Groove end projecting

Generalised equations 9 and 10 are dimensionally correct and show the change from inlet not submerged to inlet submerged occurring at HW/D values between 1.2 and 1.5. These equations are consistent with the theoretically derived equations 1 to 8.

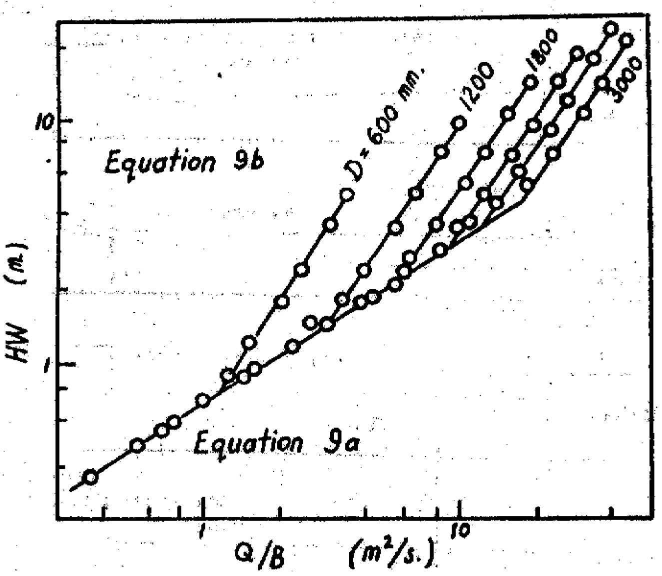


Figure 4 Equations for Box Culverts, Inlet Control

Figures 4 and 5 show equations 9 and 10 together with data points from the U.S. Department of Transportation nomographs. Agreement is very close over the complete range except at the change from inlet submerged to inlet not submerged. Here the points deviate from the equations and the equations overestimate discharge on average by 5%.

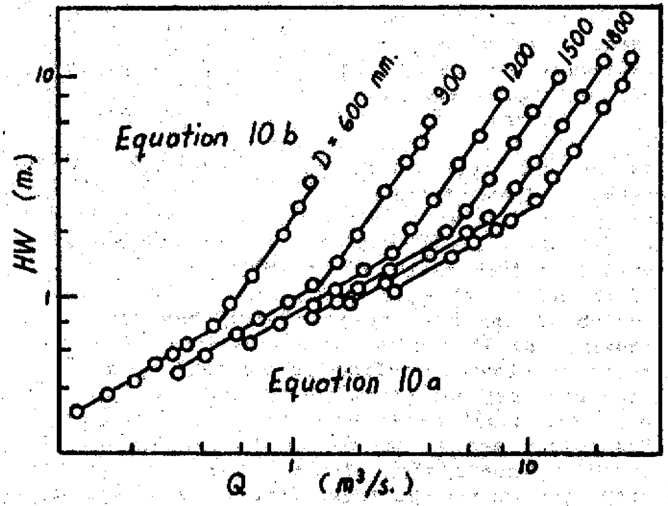
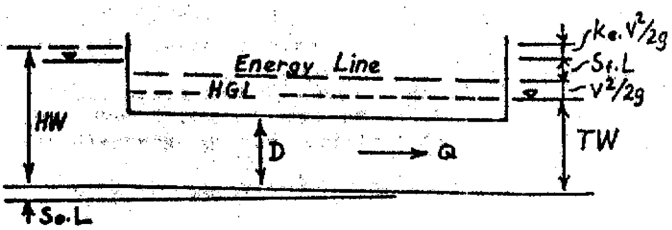


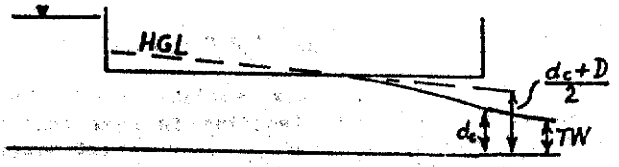
Figure 5 Equations for Circular Pipe Culverts, Inlet Control

6 DISCHARGE WITH OUTLET CONTROL

Culvert flow with outlet control may occur for high tailwater depths TW, and for culverts of large length L laid at flat slope S_o. Culvert discharges are then affected by entrance and exit losses, and friction losses in the barrel. For a culvert of total cross section area A and velocity in the barrel $v = Q/A$, the equation is derived from Figure 6a. Note that the hydraulic grade line (HGL) coincides with the water surface upstream and downstream of the culvert and is $v^2/2g$ below the energy line in the barrel.



(a) Outlet Submerged



(b) Outlet Not Submerged

Figure 6 Culvert with Outlet Control

$$HW = TW + (v^2/2g + S_f.L + k_e.v^2/2g) - S_o.L \qquad (11)$$

The energy line slope S_f is obtained from Mannings equation

$$S_f = v^2n^2/R_h^{1.333} \qquad (12a)$$

where the hydraulic radius R_h = flow cross section area/wetted perimeter, and Manning n is near to 0.011 for concrete. Alternatively, S_f can be obtained from the Darcy-Weisbach equation

$$S_f = f.v^2/8g.R_h \qquad (12b)$$

The flow in the barrel is rough turbulent if

$$\mathrm{IR}.\sqrt{f}.e/D \geq 200 \qquad (13)$$

where e = pipe roughness. This condition will usually hold for culverts in practice, and the friction factor f can then be taken as constant depending on the pipe roughness. For this case f and n are related by

$$f = 8\,gn^2/R_h^{0.333} \qquad (14)$$

The bracketed term of equation 11 contains the exit, friction and entrance losses respectively and is called the total head loss H. Values of H calculated from equation 11 agree closely with the U.S. Department of Transportation nomographs for outlet control.

Outlet control may also occur with the outlet unsubmerged (Figure 6b). Two cases are possible. For TW < d_c, critical depth occurs at the outlet and the flow depth d_c can be obtained from equation (1) or (4) for box and circular pipe culverts respectively. For d_c < TW < D the depth at the outlet equals TW. For both of these cases the headwater level is still calculated from equation (11) but with TW replaced by an equivalent hydraulic grade line level h_o at the outlet.

The equivalent hydraulic grade line h_o is taken to be the greater of $(d_c + D)/2$ and TW, with a maximum value of h_o = D. This is based on the assumption that the equivalent hydraulic grade line continues to fall linearly after the water surface leaves the culvert crown, and this has been verified by backwater calculations (U.S. Department of Transportation, 1965).

If $HW < D + (1 + k_e)v^2/2g$ the hydraulic grade line inside the culvert entrance lies below the crown, and part full flow occurs throughout the barrel. In this case the previous methods are accurate only for $HW > 0.75D$. For lower values, more accurate results can be obtained from backwater calculations or from reference (13).

As for inlet control, the HW level refers to the energy line in the headwater pond and will be equal to the headwater depth if the approach velocity $v_1^2/2g$ is negligible.

6 PERSONAL COMPUTER PROGRAMS FOR CULVERT DISCHARGES

Two computer programs utilizing the generalised equations have been written and are available from the author, Department of Civil and Mining Engineering, University of Wollongong, P.O. Box 1144, Wollongong, 2500.

Program CULVERT.HW is interactive and calculates the headwater level HW for any discharge in box or pipe culverts of any size. The program distinguishes between inlet control and outlet control and is based on the flow chart of Figure 1.

Program CULVERT.Q calculates discharges Q corresponding to a range of specified headwater levels HW, for any combination of culvert size, type and number, plus an overflow spillway placed at any level relative to the culverts.

7 CONCLUSIONS

Generalised equations, based on the U.S. Department of Transportation method have been developed for box and circular pipe culverts subject to inlet and outlet control. Equations 9 refer to box culverts with inlet control and equations 10 refer to circular pipe culverts with inlet control. Equations 11 and 12 refer to both box and pipe culverts with outlet control. Critical depth in box and pipe culverts are given by equations 1 and 4 respectively.

8 ACKNOWLEDGEMENT

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