Application of Gerris to electroconvection problems

Pedro A. Vázquez

Dpto. de Física Aplicada III

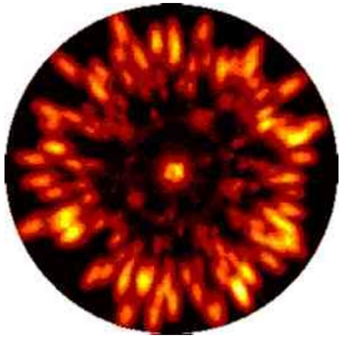
Universidad de Sevilla

Outline

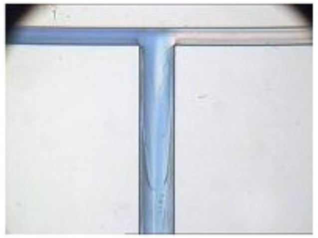
- What is Electrohydrodynamics (EHD)?
- Charge conduction mechanisms and EHD equations
- Electroconvection between parallel plates
  - Statement of the problem
  - Equations and boundary conditions
  - Gerris simulation and comparison with analytical results and other methods

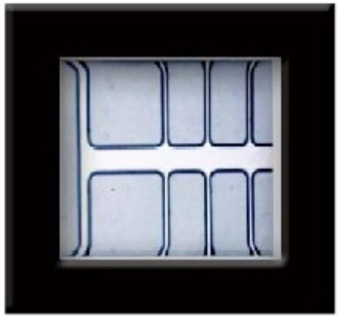
- Electrohydrodynamics (EHD) is an interdisciplinary area dealing with the interaction of fluids and electric fields and charges
- The electric charge can appear in the volume of the fluid (space charge) or on the surface interfaces between fluids (surface charge)
- The electric and velocity fields are coupled through the electrical forces acting upon the charges.

- Plasmas
  - Corona effect
  - Ozone generation
  - Electrostatic precipitators



- Pumping of liquids in MEMS

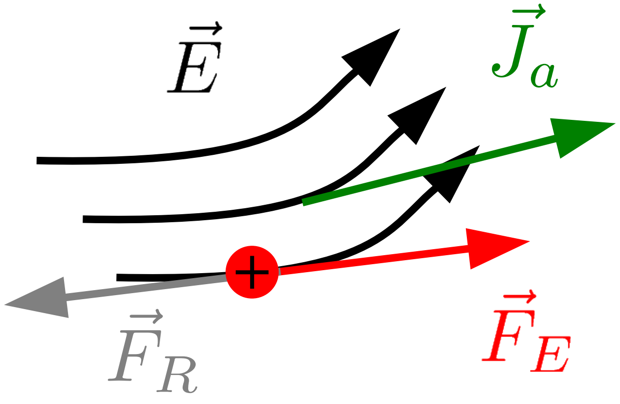




- Heat transfer enhancement

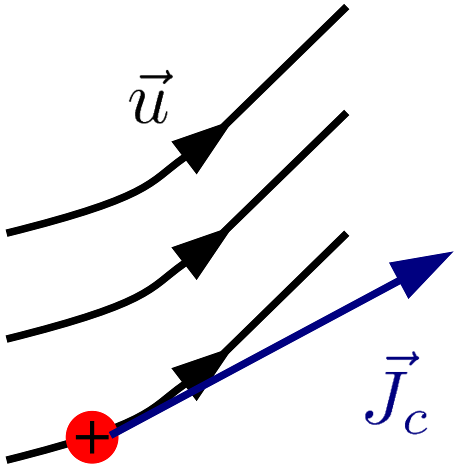
- What is Electrohydrodynamics (EHD)?
- Charge conduction mechanisms and EHD equations
- Electroconvection between parallel plates
  - Statement of the problem
  - Equations and boundary conditions
  - Gerris simulation and comparison with analytical results and other methods

Drift



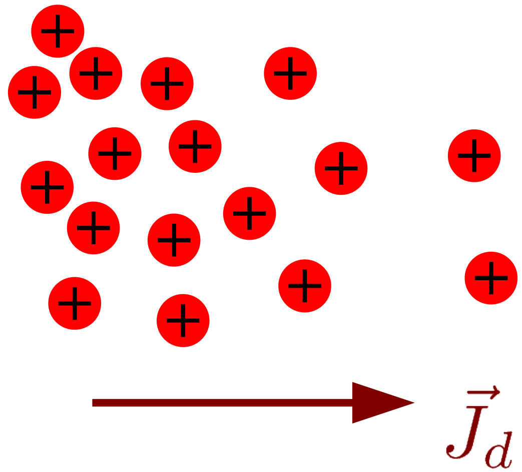
$$\vec{J}_a = K\, q\, \vec{E}$$

Convection



$$\vec{J}_c = q\, \vec{u}$$

Diffusion



$$\vec{J}_d = -D\, \nabla q$$

- Current density $\vec{J} = \vec{J}_a + \vec{J}_c + \cancel{\vec{J}_d}$
  - The diffusion current is negligible in the bulk

Electric field

$$\nabla^2 \Phi = -\frac{q}{\varepsilon} \qquad \vec{E} = -\nabla \Phi$$

Electric current

$$\frac{\partial q}{\partial t} + \nabla \cdot \vec{J} = 0 \qquad \vec{J} = q(\vec{u} + K\vec{E})$$

Hyperbolic!!!

Coupling

Hydrodynamics

$$\rho_m \frac{\partial \vec{u}}{\partial t} + \rho_m (\vec{u} \cdot \nabla)\vec{u} = -\nabla p + \eta \nabla^2 \vec{u} + q\vec{E}$$

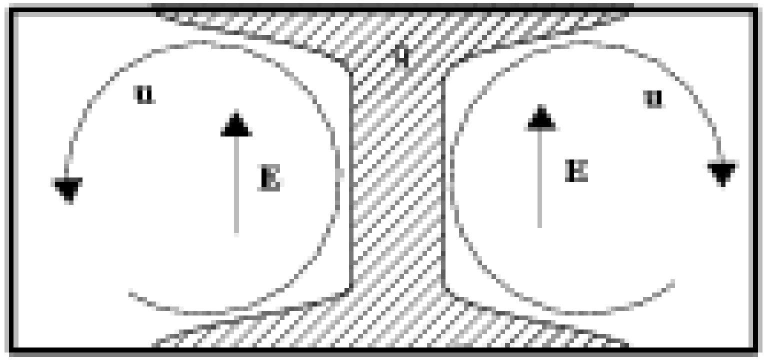
$$\nabla \cdot \vec{u} = 0$$

Outline

- What is Electrohydrodynamics (EHD)?
- Charge conduction mechanisms and EHD equations
- Electroconvection between parallel plates
  - Statement of the problem
  - Equations and boundary conditions
  - Gerris simulation and comparison with analytical results and other methods

Electroconvection between parallel plates

- Non conductive liquid between two parallel plates subjected to a electric voltage
- Above a critical threshold the bottom electrode injects electric charges in the liquid, with its same polarity



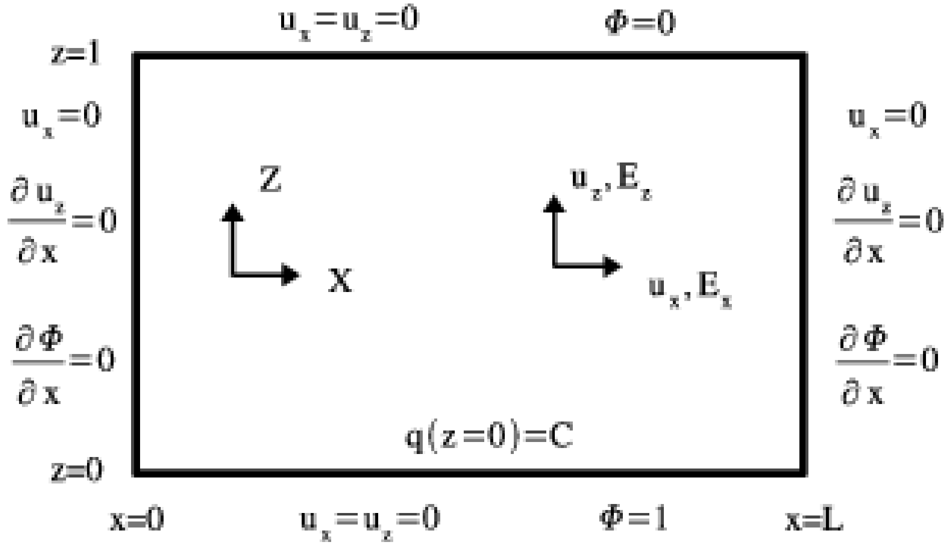
- The electric field pushes the charges away from the injecting electrode electrode
- The charges pushes the neutral molecules and all the liquid is put into motion if the applied voltage is high enough

- Non-dimensional equations and boundary conditions

$$\nabla^2\Phi = -q \qquad \vec{E} = -\nabla\Phi \qquad \nabla\cdot\vec{u} = 0$$

$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\nabla p + \frac{\mathrm{M}^2}{\mathrm{T}}\nabla^2\vec{u} + \mathrm{M}^2 q\vec{E}$$

$$\frac{\partial q}{\partial t} + \nabla\cdot[q(\vec{u}+\vec{E})] = 0$$



- Non-dimesional parameters

$$\mathrm{T} = \frac{\varepsilon\Phi_0}{\eta K}$$ Electric force / viscosity (Electric Rayleigh number)

$$\mathrm{M} = \frac{1}{K}\sqrt{\frac{\varepsilon}{\rho}}$$ Mobility

$$\mathrm{C} = \frac{q_0 d^2}{\varepsilon\Phi_0}$$ Injection strength

- There is a threshold value of the stability parameter T_c

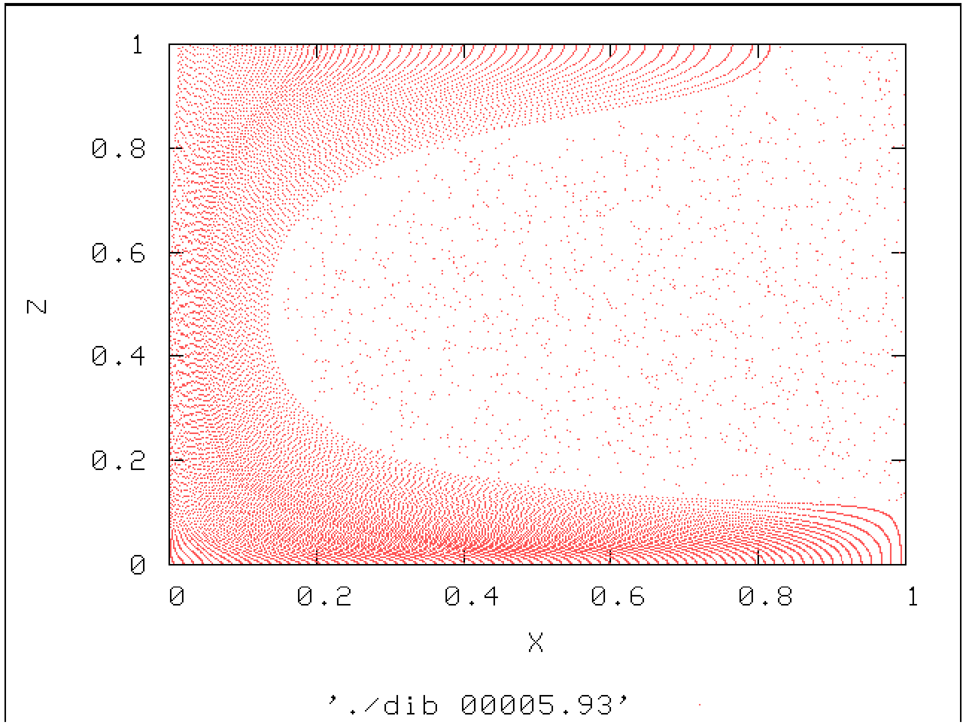
$T < T_c$ → Viscosity forbids the motion of the liquid

$T > T_c$ → A velocity roll developps with a maximum velocity greater than the maximum electric field (non-dimensional)

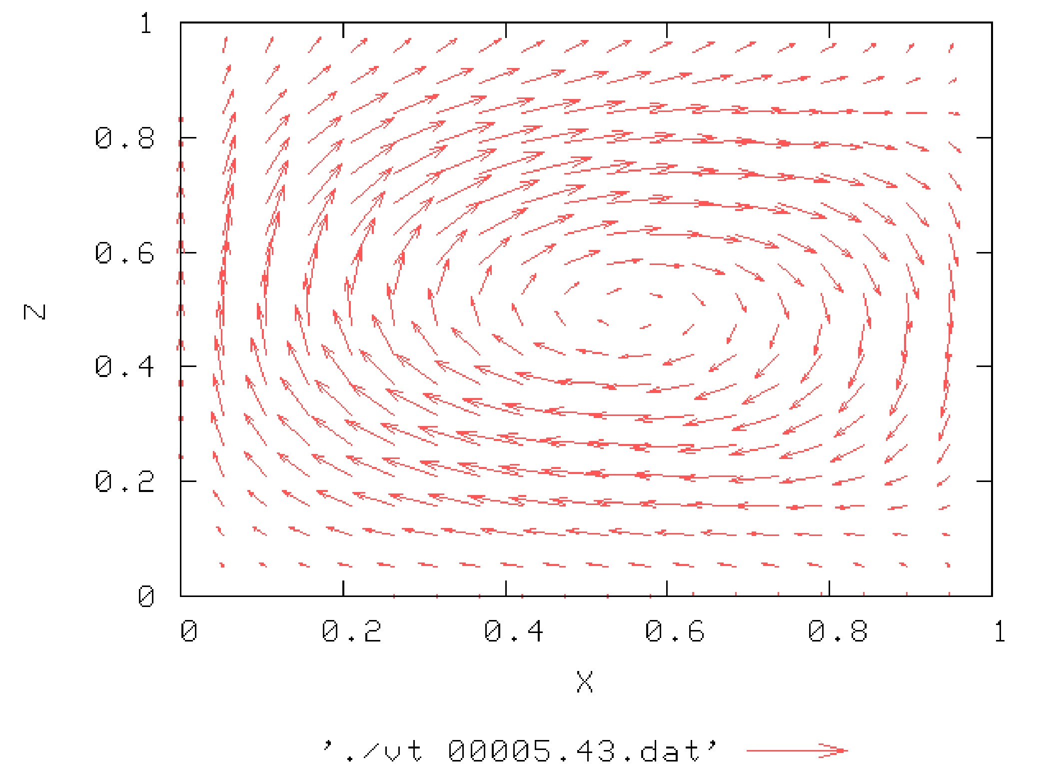
- The distribution of electric charge is controlled by the velocity of the fluid: appearence of regions with no electric charge
- The analitic linear stability analysis gives, for every value of C, a critical T_c and a critical wavelength

- Numerical simulation with Particle-In-Cell + Finite Elements in half a convective cell with $T>T_c$

Particles (charge) distribution



$$\vec{v}_P = \vec{u} + \vec{E}$$



Outline

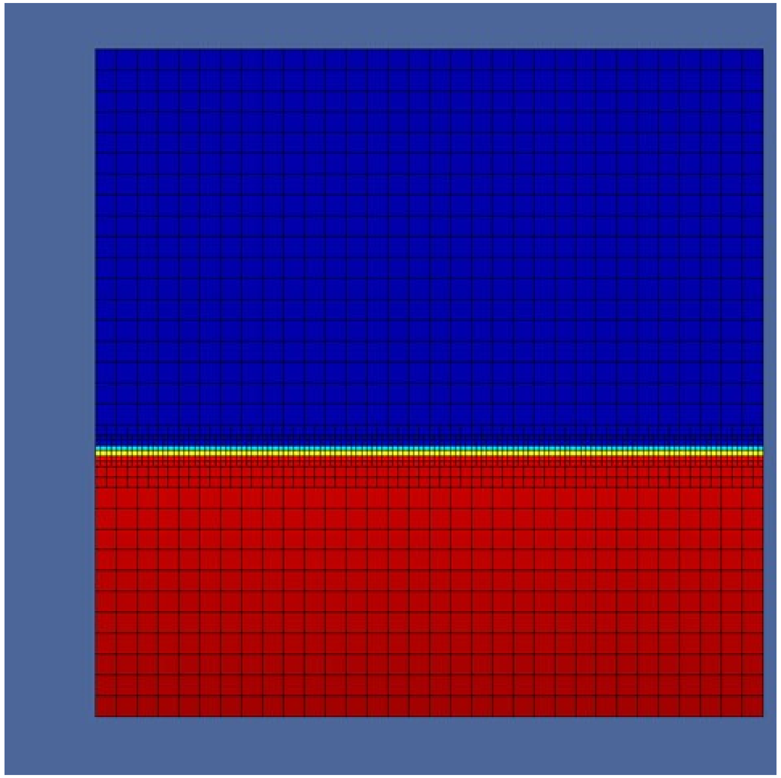
- What is Electrohydrodynamics (EHD)?
- Charge conduction mechanisms and EHD equations
- Electroconvection between parallel plates
  - Statement of the problem
  - Equations and boundary conditions
  - Gerris simulation and comparison with analytical results and other methods

Comparison with other numerical methods

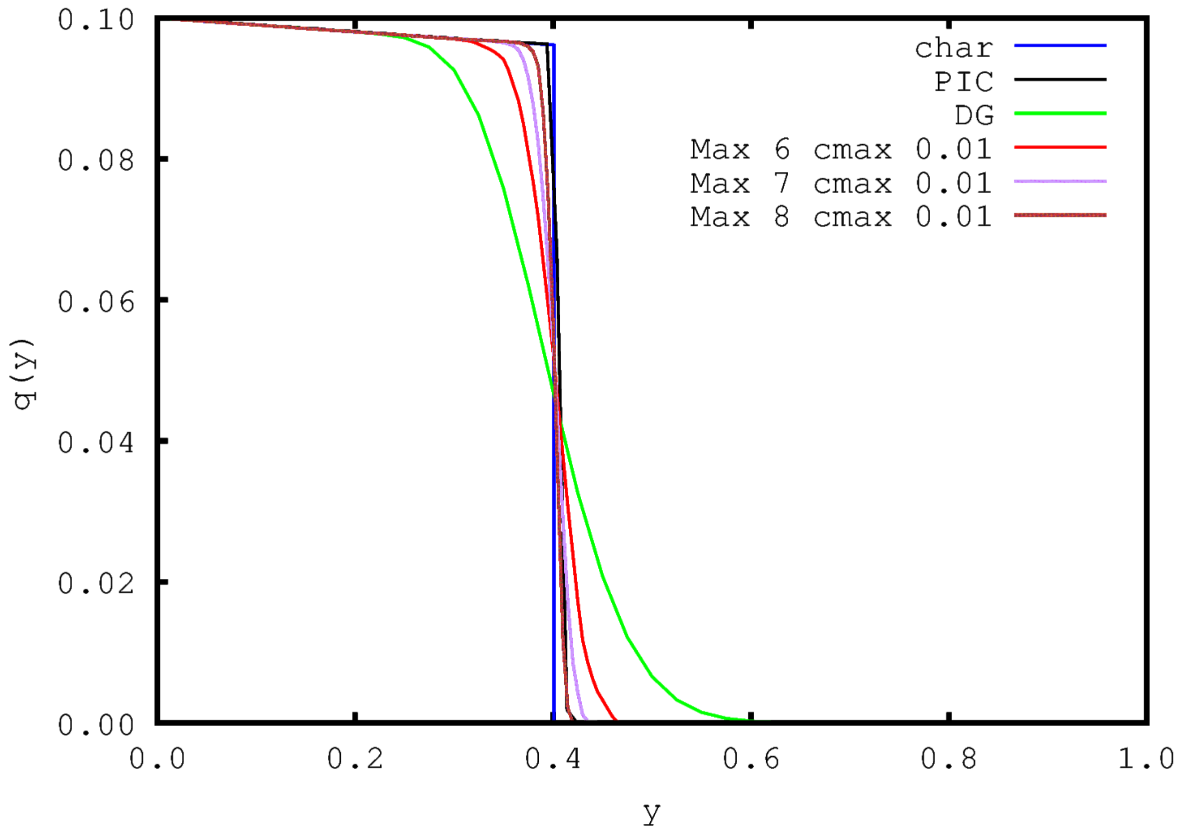
- This problem is very sensitive to numerical diffusion
  - Hyperbolic problem for charge transport
  - Stagnation point for the total ionic velocity
  - Long physical times
- Other numerical methods used in EHD
  - Characteristics (only 1D)
  - Particle-In-Cell + Finite Elements
  - Discontinuous Galerkin Finite Elements
  - Finite Volume + TVD
  - FCT + Finite Elements

- We start with no charge in the volume and with only the electric field
  - An advancing front of charge developpes until a steady state is reached
  - The aim is to test the diffusivity of the numerical method

Charge distribution (Gerris)

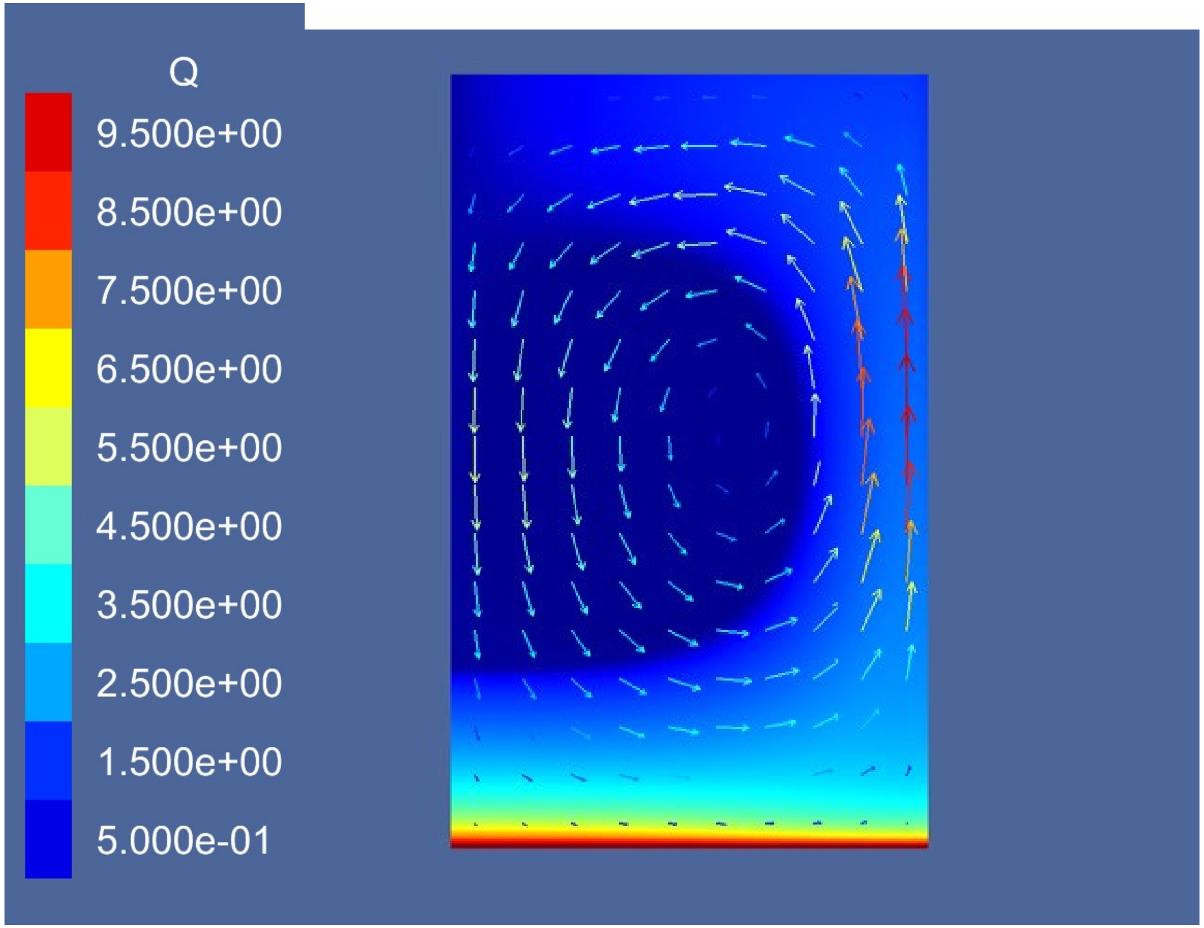


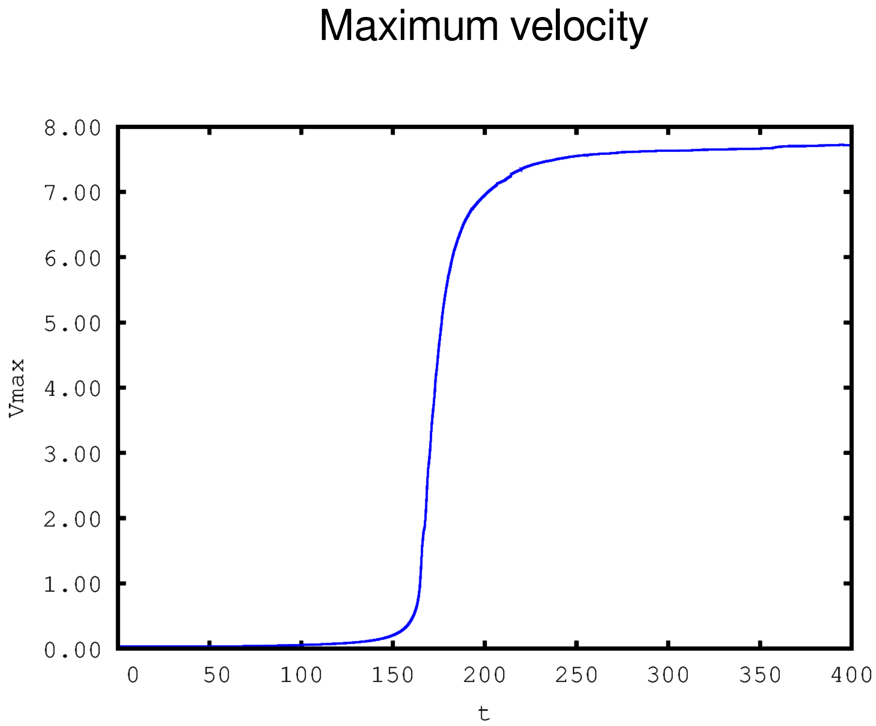
Charge along the central vertical line for t=0.4



- The simulation starts with the hydrostatic profile of charge. We fix the value of T. Then we compute the electric field and the velocity until a steady state is reached
  - If T>Tc a velocity roll developps with regions free of electric charge

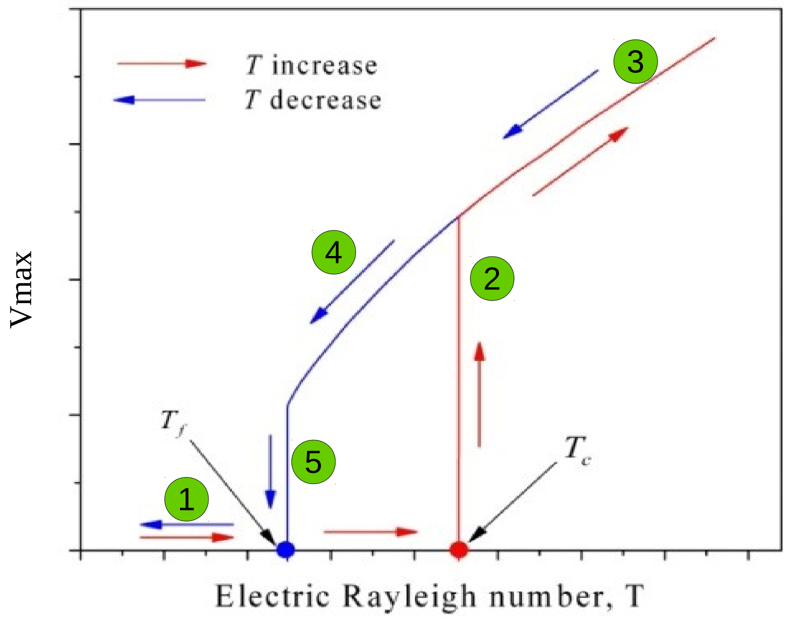
Charge distribution and velocity field with C=10 (Gerris)





- What happen in a real experiment?
- T is the non dimensional applied electric potential
- We apply an electric potential (fix T) and wait to obtain a quasi steady state

1. $T<T_c \rightarrow$ no motion
2. T increases. When $T>T_c \rightarrow$ motion
3. T further increases $\rightarrow$ max velocity increases
4. T decreases $\rightarrow$ the motion remains even for $T<T_c$
5. $T<T_f \rightarrow$ motion dissapears

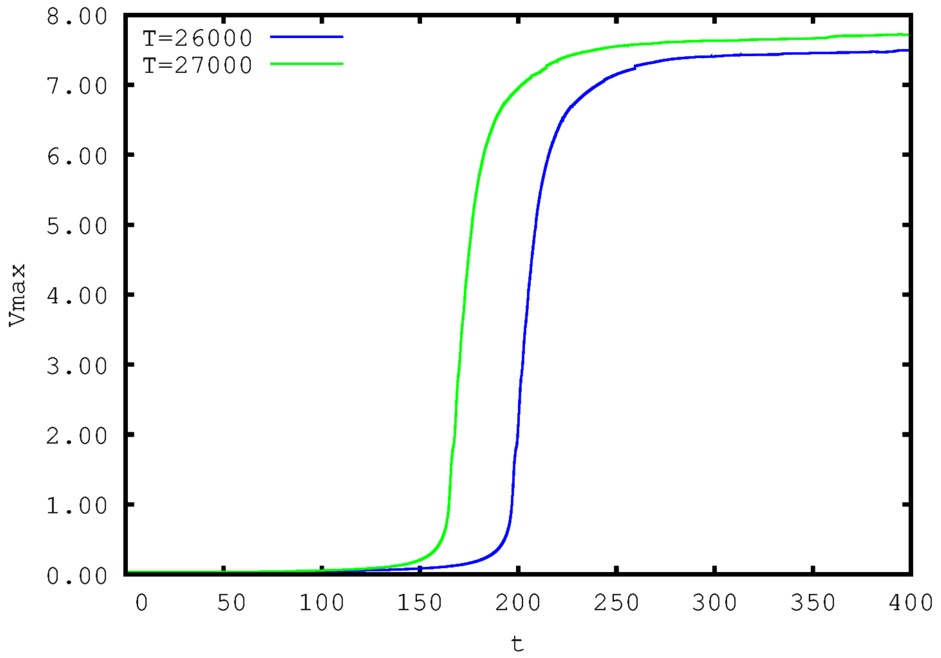


There are two critical values

T_c : linear critical threshold

T_f : non-linear critical threshold

- We can compare the linear stability criteria with the analytical values
  - The numerical value is obtained from the growth factors in the exponential regions
  - The agreement is excellent (that happens with all methods)

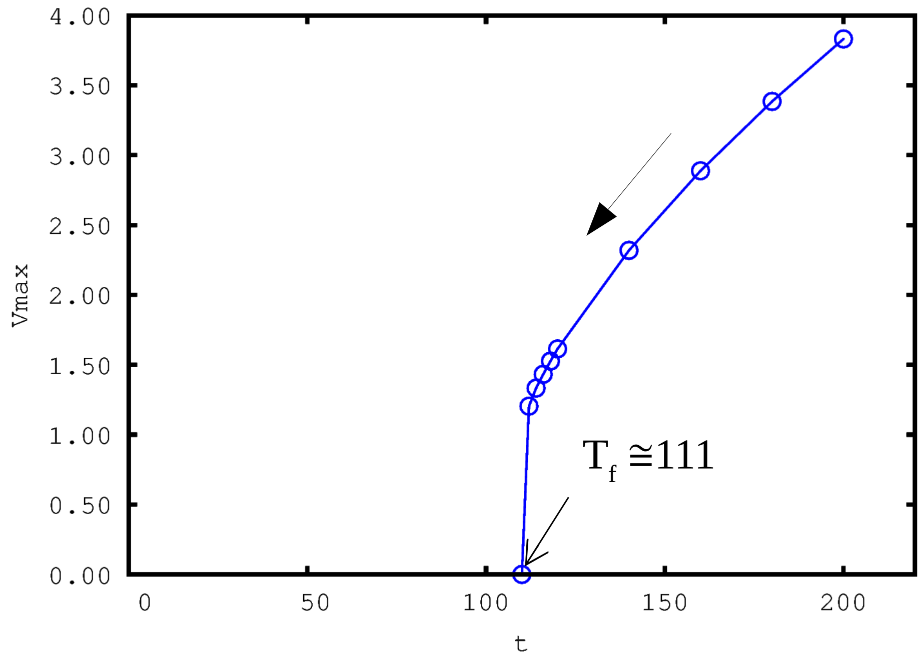


C	Tc (Analytical)	Tc (Gerris)	Dif (%)
10	164.1	165.0	0.5
0.1	24148	24000	0.6

- We can compare the linear stability criteria with the analytical values and other numerical shcemes

Values of T_f (C=10)

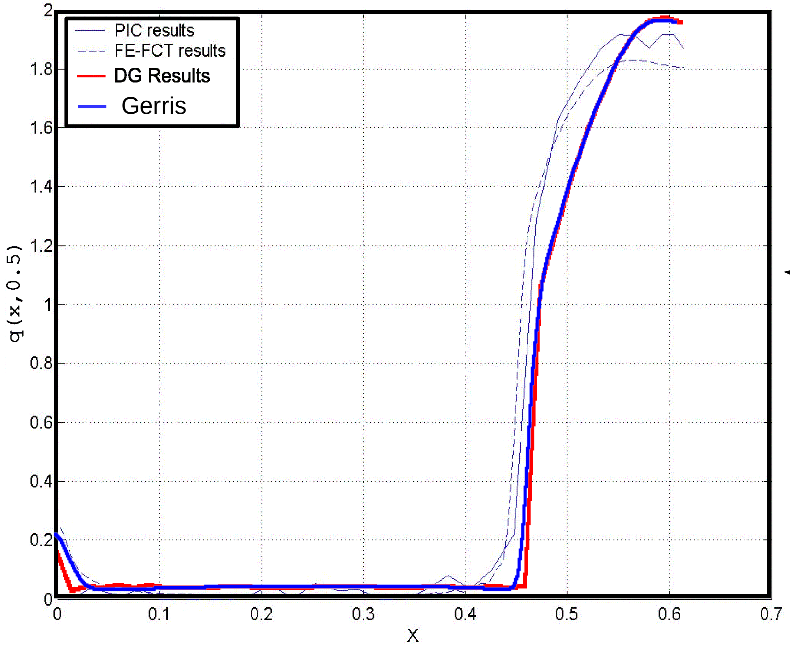
Analytical	PIC	Gerris	FV + TVD	DG
125	≅126	≅111	≅108	≅108



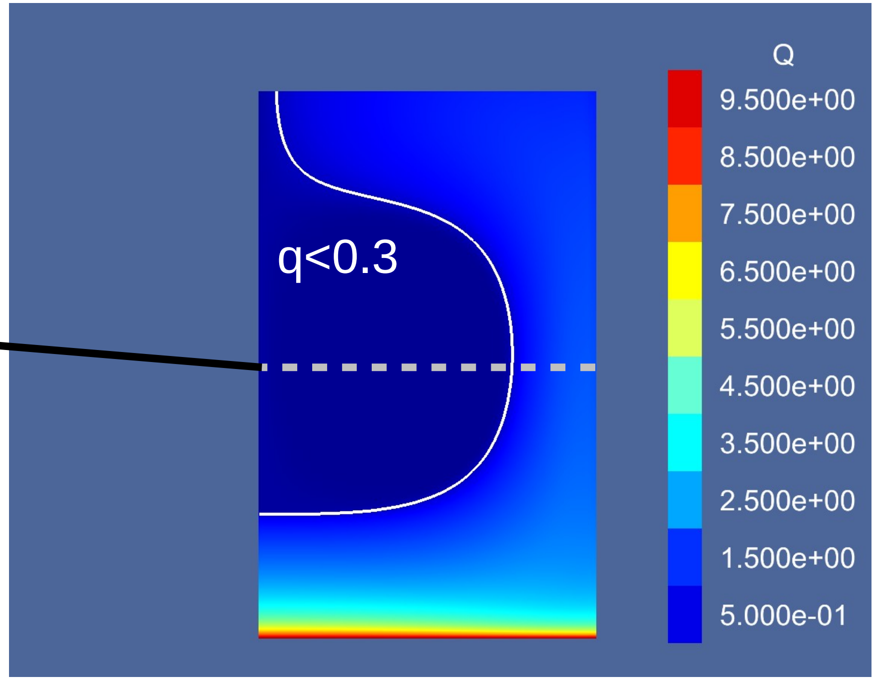
Steady state: C=10 (strong injection)

- The charge in the inner region should be zero

Charge along the central horizontal line



Charge distribution (Gerris)

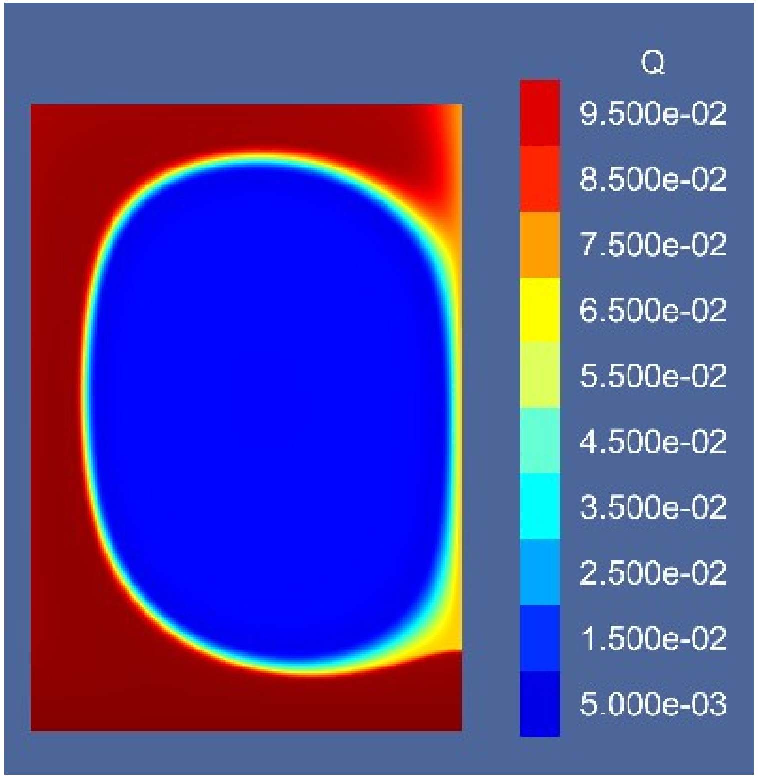


- The results with Gerris are similar to those obtained with DG and FCT, although not as good as with PIC
  - They could be improved with further refining, but more computational time

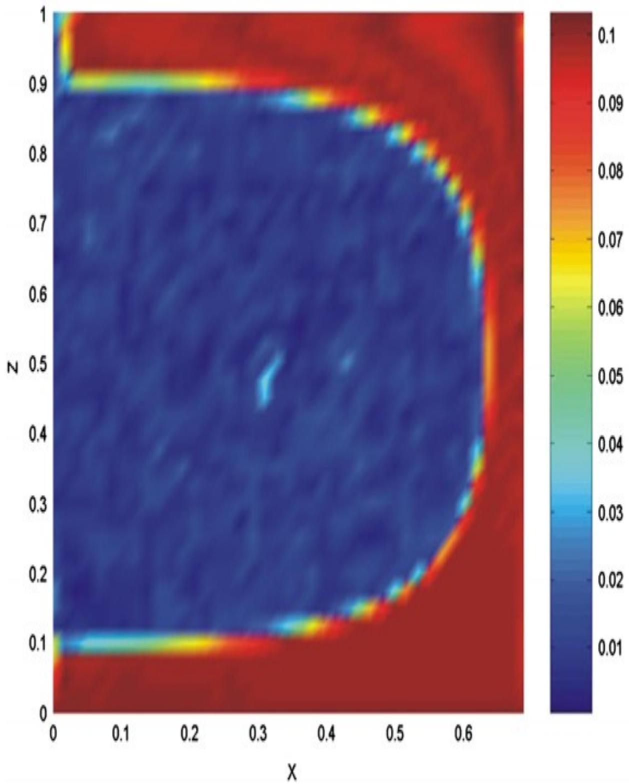
Steady state: C=0.1 (weak injection)

- The charge in the inner region should be zero

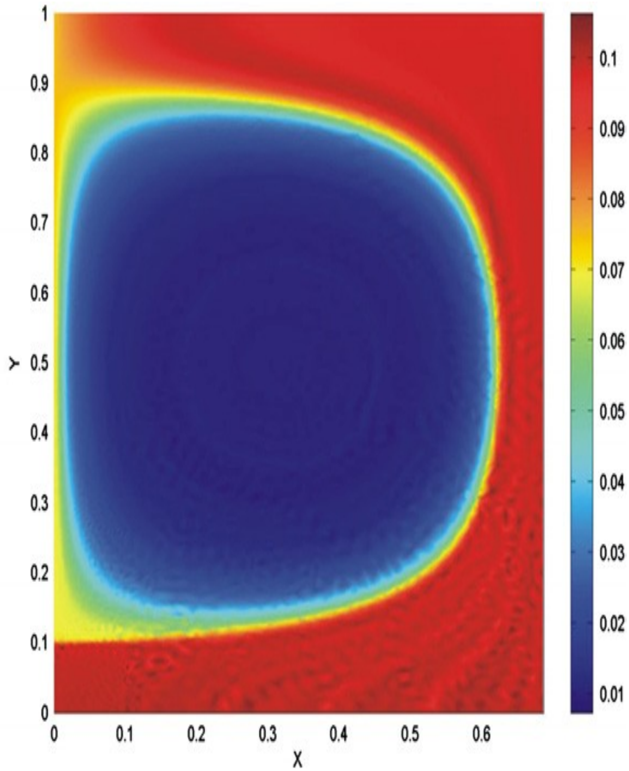
Charge distribution (Gerris)



PIC



FCT-FE



- The results with Gerris are similar to those obtained with DG and FCT, although not as good as with PIC

Summary

- Gerris results are similar to those obtained with DG, FCT-FE, FV-TVD
- In some conditions PIC-FEM is still better, but it has its own problems (computational costs, implemetation of injection condition, parasite oscillations)
- Strengths of Gerris :
  - Very competitive in terms of computational time (adaptive meshing)
  - Parallelized
  - 3D ready
- Things to try/add:
  - Addition of injection law
  - Computation of electric currents (convective + displacement)
  - 3D case (the real one!)

Thank you for your attention