Hydrodynamics of Lakes and ecosystems

Jair REYES

Institut Jean le Rond d'Alembert - UPMC
Doctoral Advisor: Maurice ROSSI

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Introduction

The role of hydrodynamics on the dynamics of algae populations in lakes.

- What are the determining factors in the explosive growth of certain algae ?

This thesis is divided into two parts

- Analysis of the fluid dynamics in lakes (Presented here).
- Coupling biology models with hydrodynamics models

Content

1.- Comparison Hydrostatic and Nonhydrostatic model

2.- Sediment resuspension using GERRIS code.

2.a The model and its validations

2.b Example of sediment resuspension due to currents

2.c Example of sediment resuspension due to Internal Solitary Waves breaking

Hydrodynamics of Lakes
Physical Models

Hydrostatic model (FORTRAN Code):

- Model widely used in marine science
- Vertical momentum equation replaced by the hydrostatic approximation.
- Vertical velocity calculated from the continuity equation.
- Primitive equations solved by using sigma coordinates

Is the hydrostatic model correct near boundaries?

Hydrodynamics of Lakes
Physical Models

Nonhydrostatic model (GERRIS Code):

- Navier-Stokes equations
- Effective viscosity
- Surface: Rigid-Lid Approximation

We have compared the hydrostatic and nonhydrostatic model.

Hydrodynamics of Lakes

Different simplified topographies: flat and parabolic basins.
We are interested in

- Structure of currents
- Shear on the bottom

Surface: $\left(\mu \frac{\partial u}{\partial z}, w\right) = (\tau_s, 0)$

Walls: Slip condition

Bottom: No-slip condition

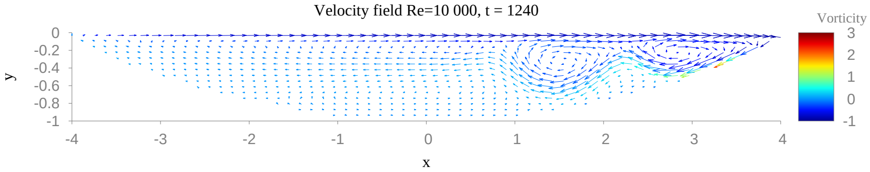
Different cases

- Differents Reynolds numbers: $\mathrm{Re} = \frac{\rho[U][L]}{\mu} = \frac{\rho \tau_s H^2}{\mu^2}$.

H [m]	μ $[\frac{\mathrm{kg}}{\mathrm{ms}}]$	U_{wind} $[\frac{\mathrm{km}}{\mathrm{h}}]$	Re	$\mathrm{Re}_{\mathrm{max}}$
10	1	3.2	100	24.0
		6.4	400	88.7
		10.3	1 000	198.5
		17.8	3 000	493.6
		32.5	10 000	1213.3

- With $\mathrm{Re}_{\mathrm{max}}$ as a Reynolds number based in the highest horizontal velocity u_{max}.

Unsteady flow

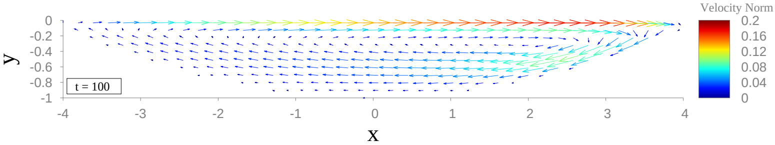


Video $\mathrm{Re} = 10000$

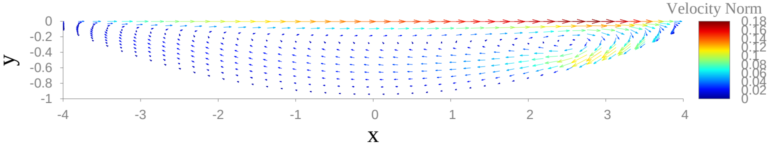
Steady velocity field

Steady velocity field corresponding to parabolic lake of $\mathrm{Re} = 10^3$

NH-M



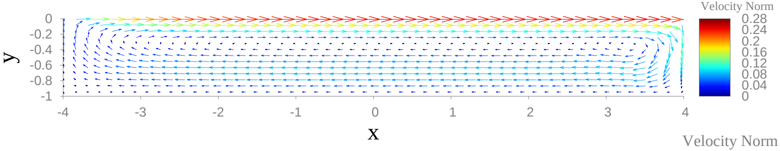
H-M



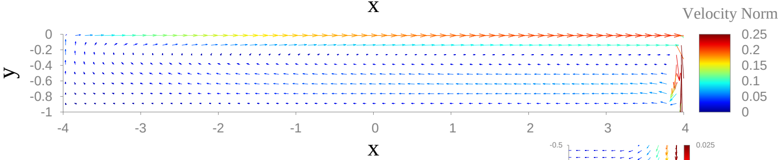
Steady velocity field

Steady velocity field corresponding to rectangular lake of $\mathrm{Re} = 4 \cdot 10^2$

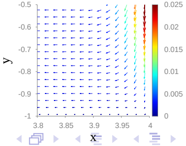
NH-M



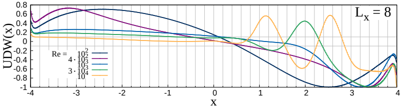
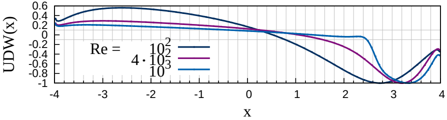
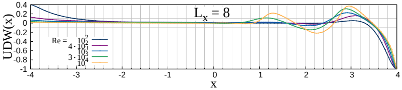
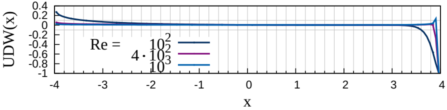
H-M



Zoom H-M



Upwelling and downwelling regions

	NH model	H model
P		
R		

Upwelling and Downwelling Function

$$\bar{w}(x) = \frac{1}{\text{top}-\text{bottom}} \int_{z=\text{bottom}}^{z=\text{top}} w(x,z)\, \mathrm{d}z.$$

Modeling Sediment

Dimensionless sediment equation

$$\phi_t + \mathbf{u} \cdot \nabla \phi = \frac{1}{\mathrm{ScRe}} \Delta \phi, \qquad (1)$$

with $\mathrm{Sc} = \frac{\nu}{D_\phi}$

Dimensionless boundary conditions:

- Surface and walls: $\frac{\partial \phi}{\partial n} = 0$
- Bottom: Suspension and deposition depending on shear. $\frac{\partial \phi}{\partial n} = -[E_b(\tau_b) + D_b(\tau_b)\phi]$.

where τ_b is the shear stress on the bottom

Modeling Sediment

Bottom boundary condition

Erosion of Sediment:

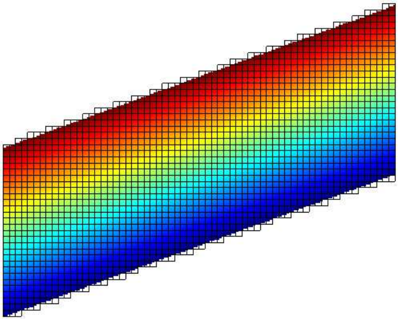
$$E_b = \begin{cases} 0 & |\tau_b| < \tau_{ce} \\ e_b \left(\frac{|\tau_b|}{\tau_{ce}} - 1 \right) & |\tau_b| \geq \tau_{ce} \end{cases},$$

Deposition of Sediment:

$$D_b = \begin{cases} 0 & |\tau_b| > \tau_{cd} \\ d_b \mathbf{n} \cdot \mathbf{e}_z \left(1 - \frac{\|\tau_b\|}{\tau_{cd}} \right) & |\tau_b| \leq \tau_{cd} \end{cases},$$

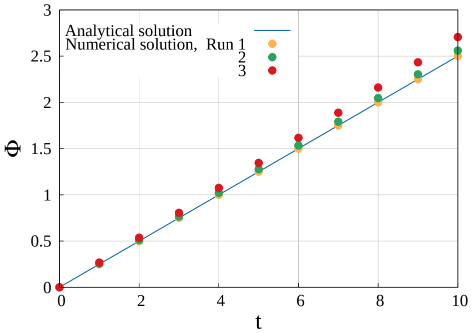
Validation: Two dimensional Inclined Couette Flow

Differents angles $\theta =$ 0,10, 20



2D Couette Flow

Sediment computation



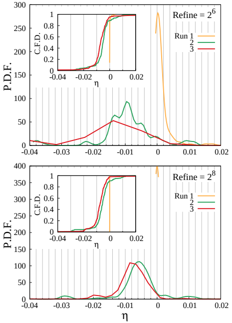
Sediment mass in the total volume

$$\Phi = \int \phi \, \mathrm{d}\mathcal{V} = \tau_s t.$$

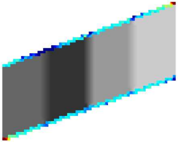
2D Couette Flow

PDF of relative error in shear computation:

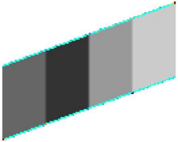
$$\eta = \frac{\tau_{s_{\text{analitic}}} - \tau_{s_{\text{numeric}}}}{\tau_{s_{\text{analitic}}}}$$



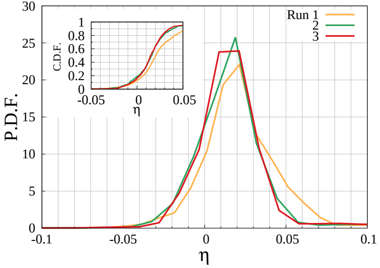
$\theta = 20$, Refine$=2^6$



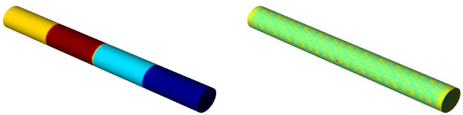
$\theta = 20$, Refine$=2^8$



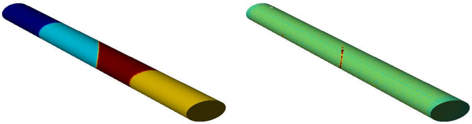
3D Poiseuille Flow



$\theta \approx 0$, Refine$=2^8$

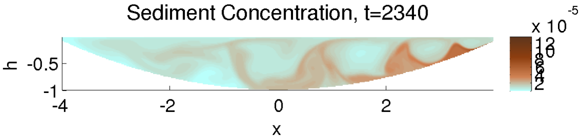


$\theta \approx 40$, Refine$=2^8$



Errors located at the MPI boundaries

Erosion of sediment in the lake



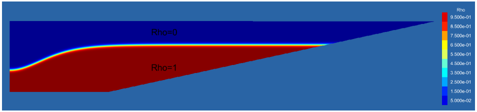
Video $\mathrm{Re} = 10^5$

Internal solitary wave (ISW) modeling

1. Momentum equation with Boussinesq approximation
2. Advection-diffusion equation for temperature

Boundary conditions

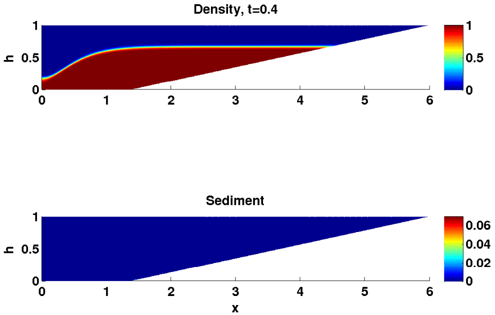
- Surface: $\left(v, \frac{\partial u}{\partial x}, \frac{\partial \rho}{\partial y}\right) = (0,0,0);$
- Right wall: $\left(u, \frac{\partial v}{\partial y}, \frac{\partial \rho}{\partial x}\right) = (0,0,0);$
- Bottom: $\left(u, v, \frac{\partial \rho}{\partial n}\right) = (0,0,0).$



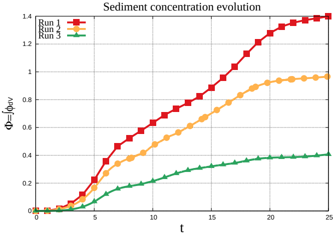
G. Rickard,J. Callaghan, S. Popinet, OMJ (2009).

ISW Breaking

Suspension of sediment du to ISW breaking and run-up



Video suspension



Sediment

$$\Phi = \int \phi \, \mathrm{d}V$$

Conclusion

- Comparison of the Hydrostatic and Nonhydrostatic Models
- Sediment resuspension du to currents in the lake.
- Sediment resuspension du to ISW breaking and run-up.