# Numerical simulations of multiphase and turbulent flows

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  - Subgrid models (particle models)

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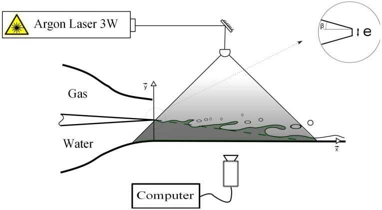


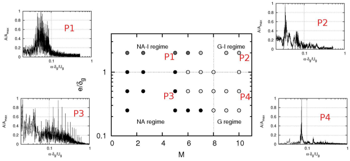

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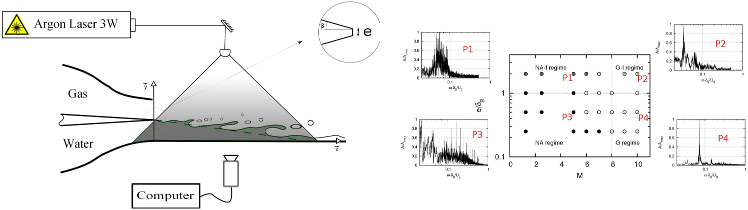


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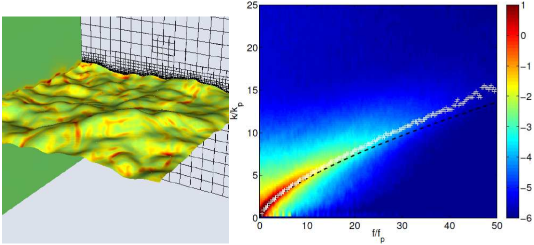

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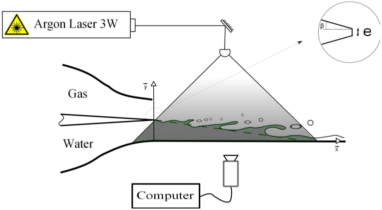


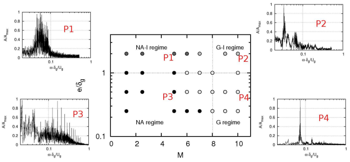


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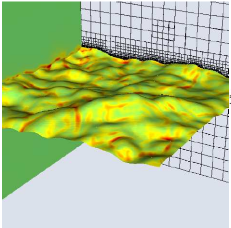

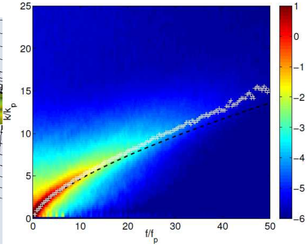


- Turbulent and reactive flows in catalytic converters Ozhan et al (CES, 2014)

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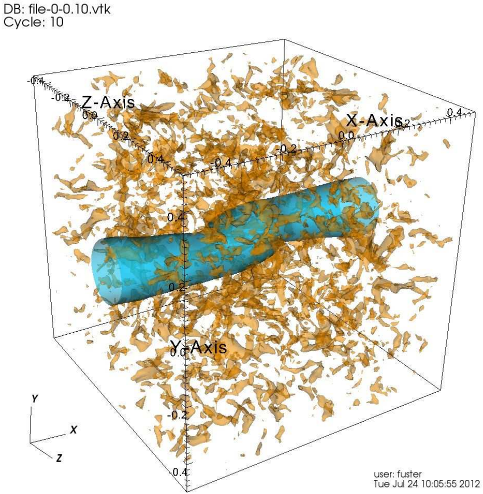
# Numerical schemes for turbulent flow simulations

How to handle multiphase flows and turbulence in Gerris?



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# Skew-symmetric formulation [J. Comp. Phys, 2013]

## Isotropic turbulence

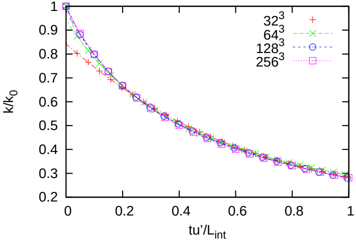
$$E(k) = \alpha \epsilon^{2/3} k^{-5/3} f_L(kL_{\text{int}}) f_\nu(kL_{\text{int}} \text{Re}_L^{-3/4})$$

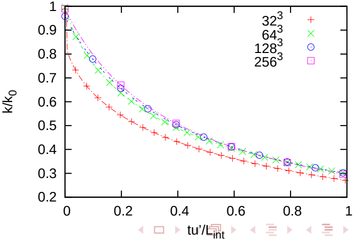
$$L_{\text{int}} = 0.5 L_{\text{box}}$$

$$\text{Re}_L = \frac{\sqrt{\int_0^\infty E(k)dk} L_{\text{int}}}{\nu} = 375$$

$$k = \int_0^\infty E(k)dk = 0.5,$$

Skew-Symmetric vs. Bell-Cotella-Glaz



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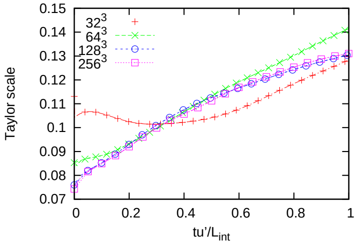


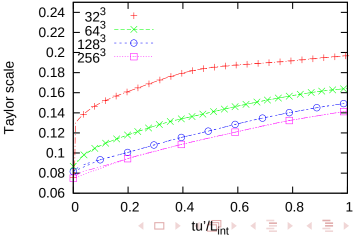

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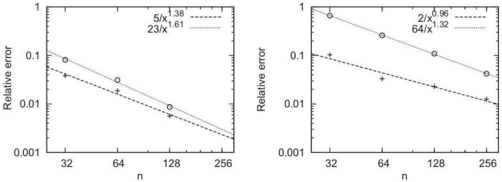
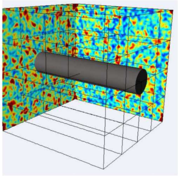
We significantly reduce the error of high order statistics

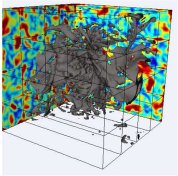



(a) Skew Symmetric (b)BCG

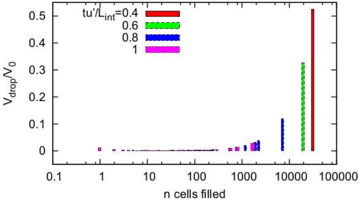
# Effect of multiple phases (same Reynolds in both phases)

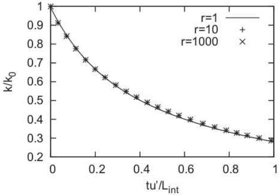


(a) $tu'/L_{\mathrm{int}} = 0$

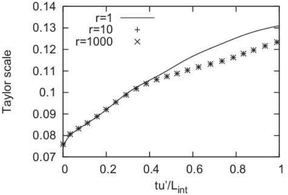


(b) $tu'/L_{\mathrm{int}} = 1$




(a) Kinetic energy




(b) Taylor microscale

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  **Work in progress**

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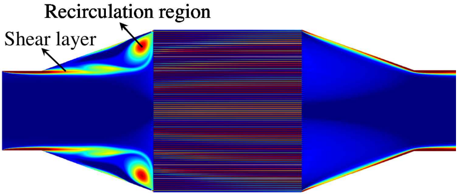
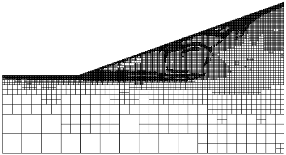
We are interested in developing numerical tools for the simulation of turbulent flows.

In particular this work is motivated by the flow simulation inside a catalytic converter system:



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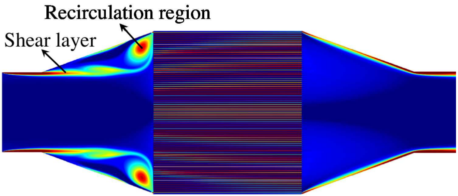


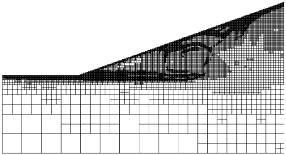

Adaptive Mesh Refinement (AMR) help us to save a significant amount of computational time!

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- What we want is to get as close as possible to the real solution trying to optimize the grid distribution

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- We want to measure the *global* and *local* efficiency of the method

We need to chose:

- A method to estimate the error
- A norm to adapt the grid:

$$\text{AMR criteria} \rightarrow \text{Error} \times \Delta x^n$$

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  - Variables related to turbulent flows (vorticity, helicity...)

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- **A residual based method**
  We try to obtain a measure of the discretization error when solving a given equation

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  - and reaction limit

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For each cell the mechanism controlling the error can be different depending on the local Reynolds number, etc....
This criteria also reduces the error propagation (we reduce the error of the RHS of the equation)

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  - Helicity equations

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  - Navier-Stokes (with velocity as primitive variable)
  - Vorticity equation
  - Helicity equations
  - Kinetic energy equation

Because of their efficiency, we compare different a-posteriori error estimation methods:

- **A residual based method**
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  - Navier-Stokes (with velocity as primitive variable)
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  - ....

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  Which quantity?

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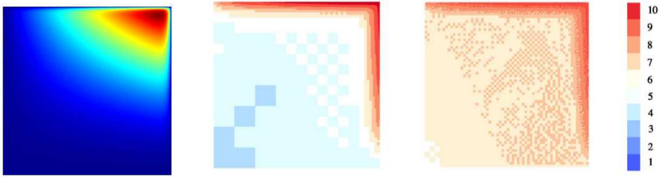
The optimal choice is problem dependent

Error estimation for the transport equation: Propagation error only controls in regions where the error is small
Steady problem



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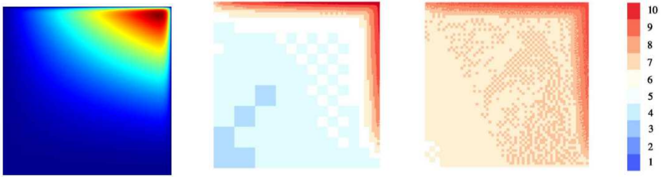
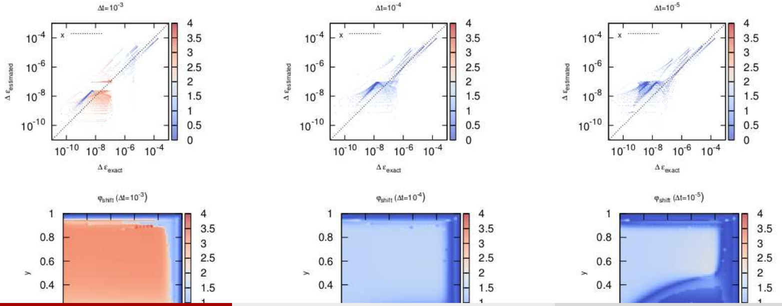
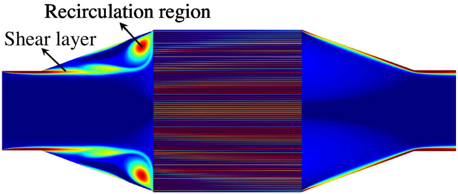
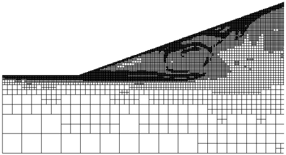



Transient $T = T_{steady} + T(t)e^{i\omega t}$



Three test cases related to our problem of interest:



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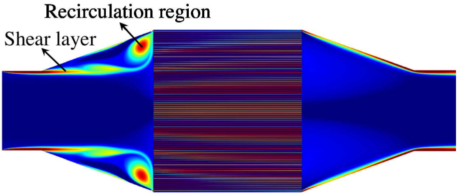


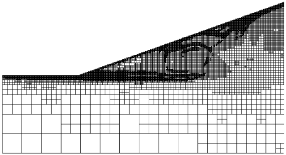

## Test cases

- Dissipation of the Lamb-Oseen vortex

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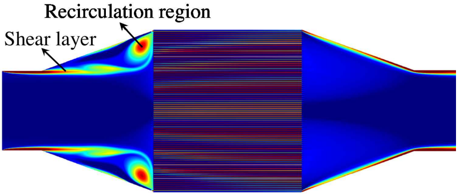


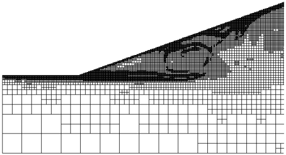

## Test cases

- Dissipation of the Lamb-Oseen vortex
- Linear growth of random noise in a shear layer

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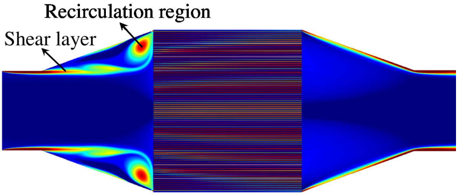
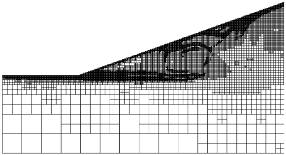
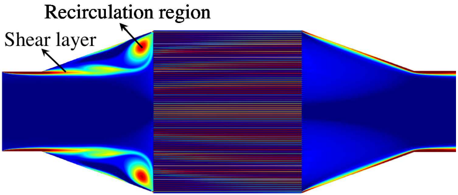


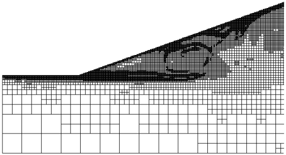



## Test cases

- Dissipation of the Lamb-Oseen vortex
- Linear growth of random noise in a shear layer
- Isotropic turbulence test case

We choose three test cases related to our problem of interest:



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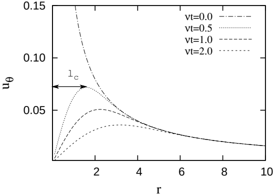
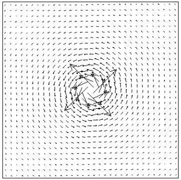
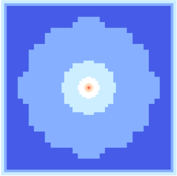
# Lamb-Oseen vortex

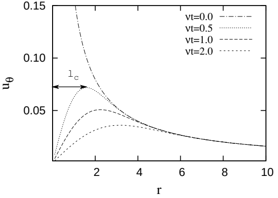


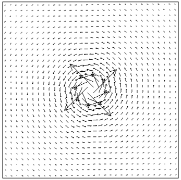



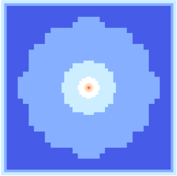


$$u_\theta = \frac{\Gamma}{2\,\pi\,r}\left[1 - exp\left(-\frac{r^2}{4\,\nu\,t}\right)\right].$$

# Lamb-Oseen vortex





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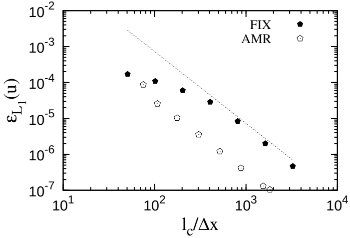
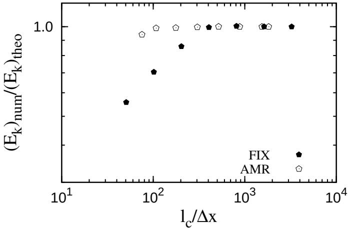
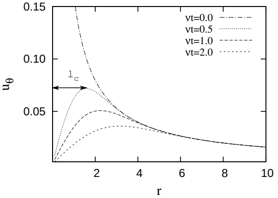
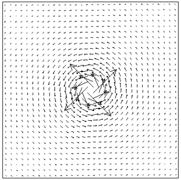
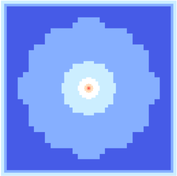


# Lamb-Oseen vortex





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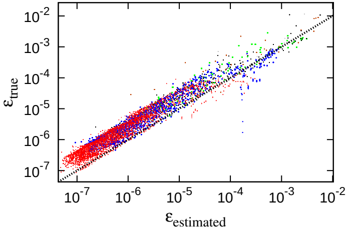
Local efficiency: Estimated error vs. Real error



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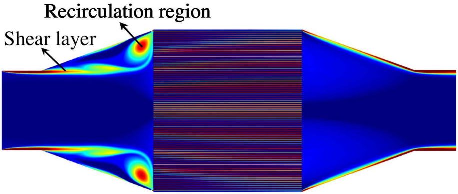
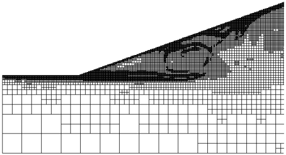


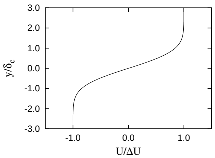



## Test cases

- Dissipation of the Lamb-Oseen vortex
- **Linear growth of random noise in a shear layer**
- Isotropic turbulence test case

## Noise growth in a shear layer

$$U = \Delta U erf\left(y/\delta_c\right)$$



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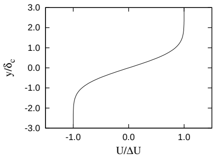


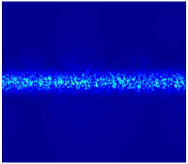

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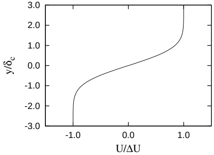


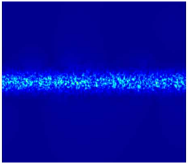

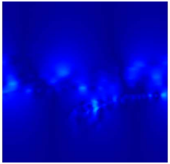

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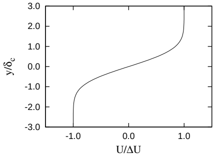


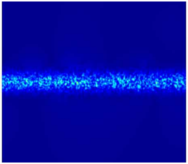

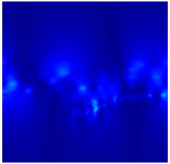

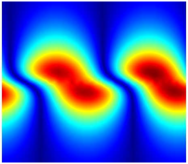

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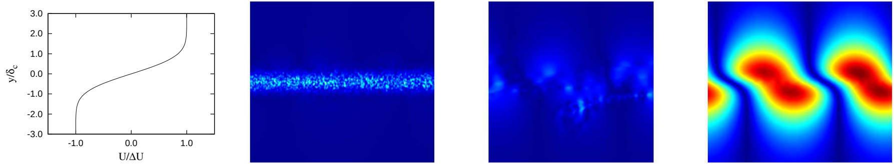


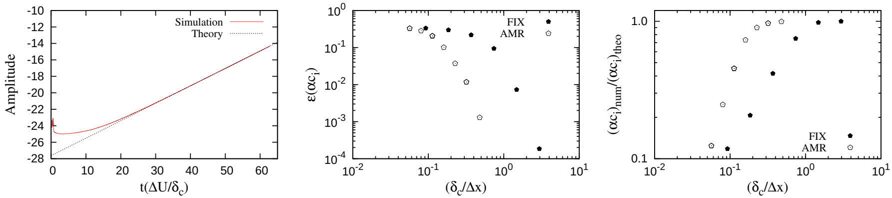

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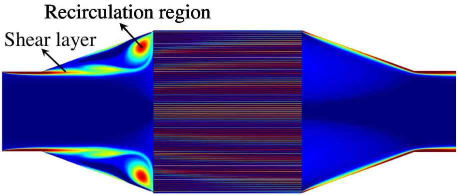
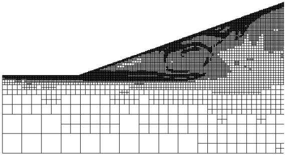


## Test cases

- Dissipation of the Lamb-Oseen vortex
- Linear growth of random noise in a shear layer
- **Isotropic turbulence test case**

# Isotropic turbulence

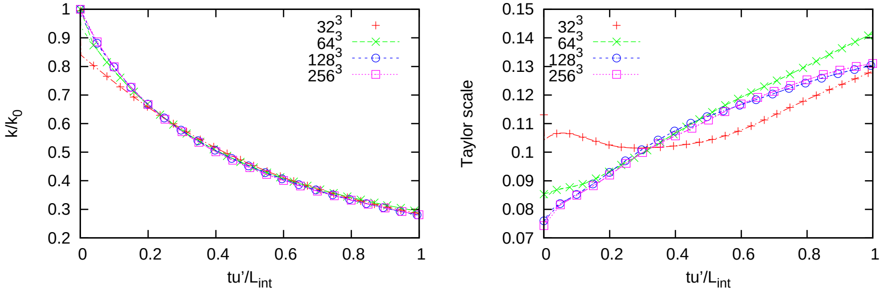
$$E(k) = \alpha \epsilon^{2/3} k^{-5/3} f_L(kL_{\text{int}}) f_\nu(kL_{\text{int}} \text{Re}_L^{-3/4})$$

$$L_{\text{int}} = 0.5 L_{\text{box}}$$

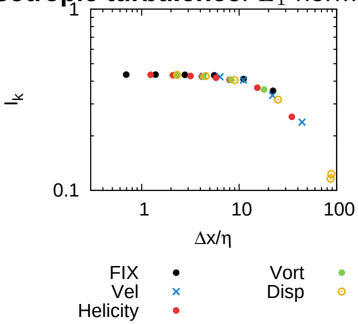
$$\text{Re}_\text{L} = \frac{\sqrt{\int_0^\infty \text{E(k)dk}} \text{L}_{\text{int}}}{\nu} = 375$$

$$k = \int_0^\infty E(k)dk = 0.5,$$

The characteristic lengthscale changes in time



## **Isotropic turbulence**: $L_1$ norm, Hessian based error estimator



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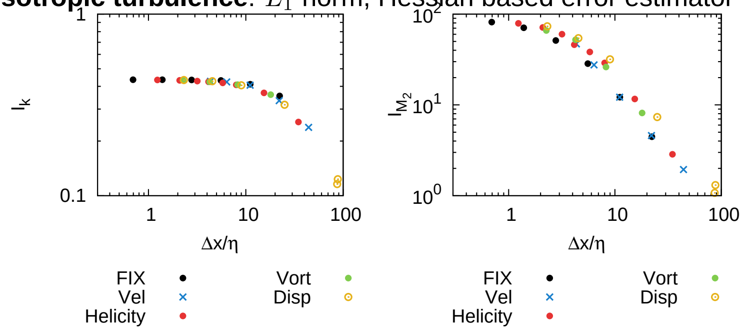

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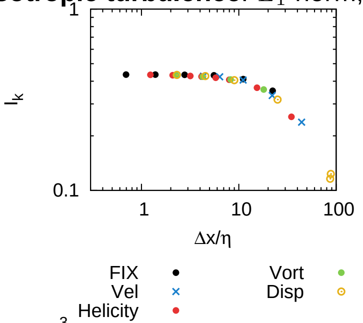


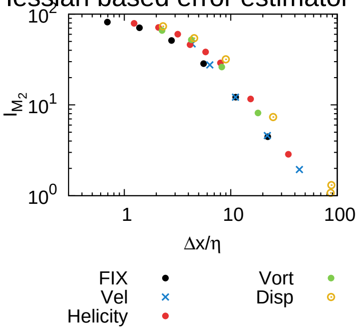


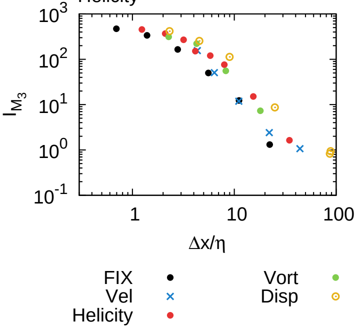

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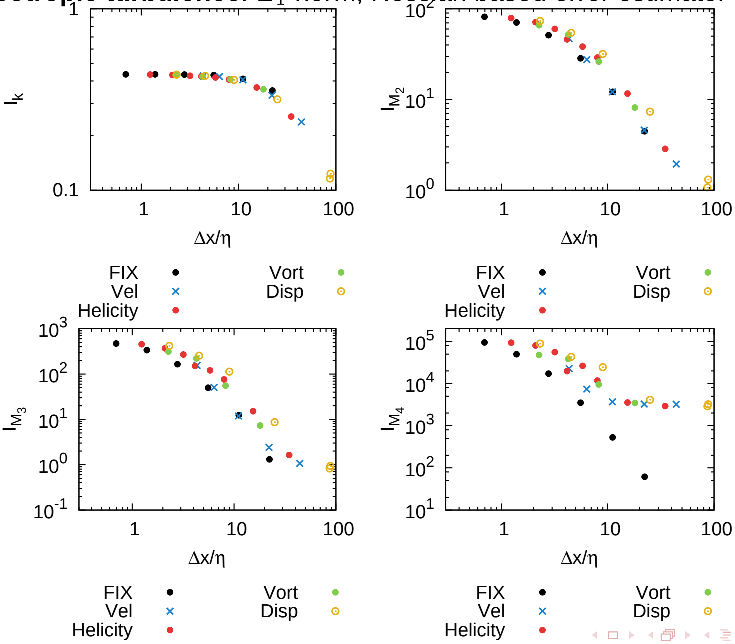
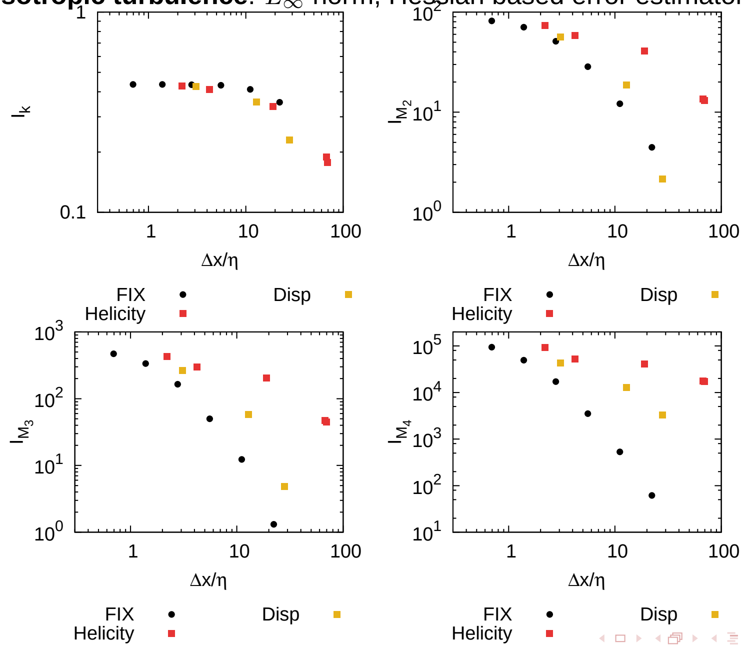
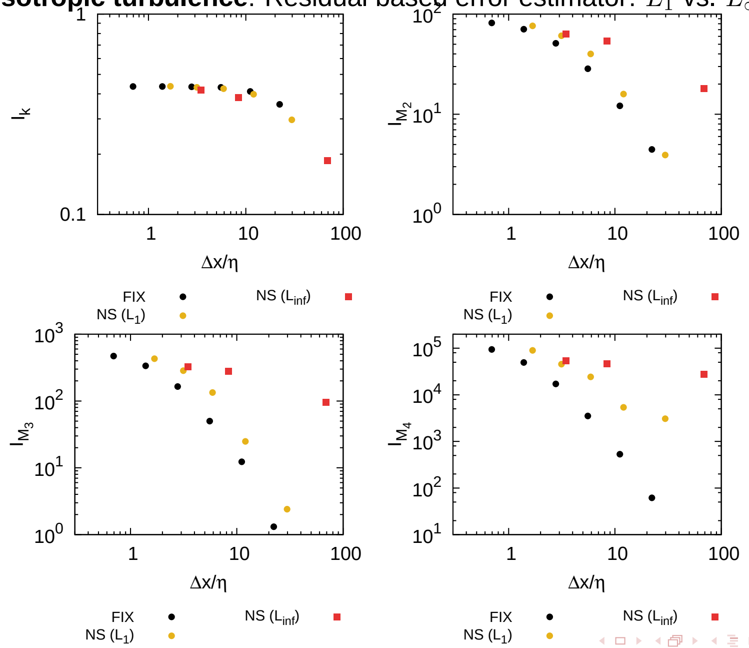


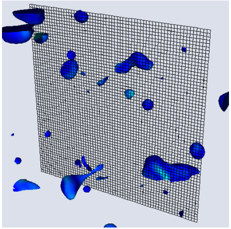
## **Isotropic turbulence**: $L_\infty$ norm, Hessian based error estimator



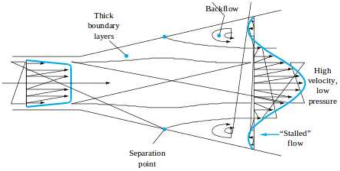
## **Isotropic turbulence**: Residual based error estimator: $L_1$ vs. $L_\infty$ norms



Perspectives: To investigate the process of bubble breakup in forced isotropic turbulence



**Experimental validation**: Benjamin [2002, Experiments in Fluids]
We look at the distribution of flow through a catalytic converter:

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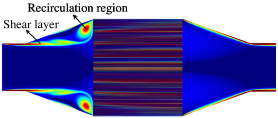

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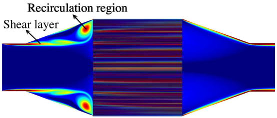


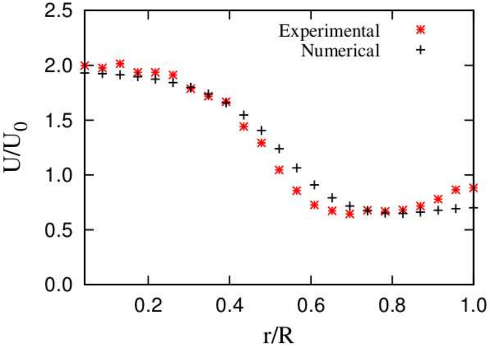

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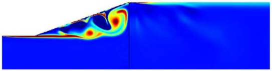
$$\sigma_V = \frac{1}{\dot{m}} \int_A |U_i - U_e| \, \delta \dot{m}$$

**Re = 20000**



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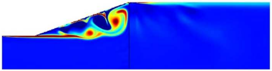

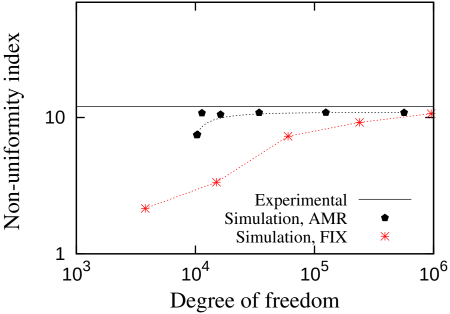

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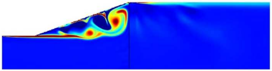
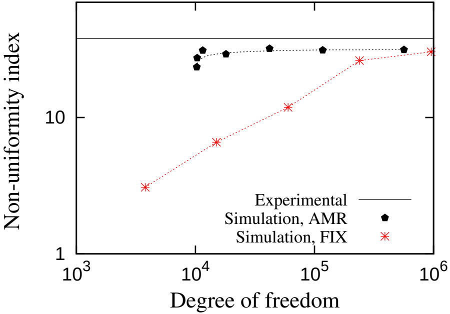
**Re = 60000**





**Experimental validation**: Benjamin [2002, Experiments in Fluids]
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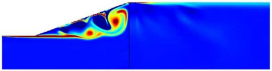
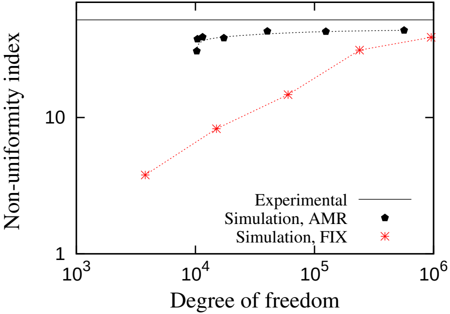
$$\sigma_V = \frac{1}{\dot{m}} \int_A |U_i - U_e| \, \delta\dot{m}$$

**Re = 80000**





**Experimental validation**: Benjamin [2002, Experiments in Fluids]
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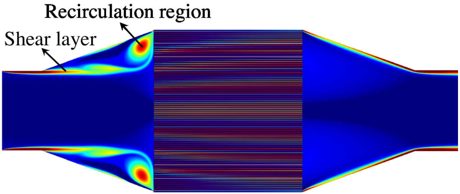

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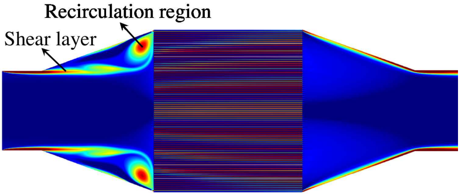


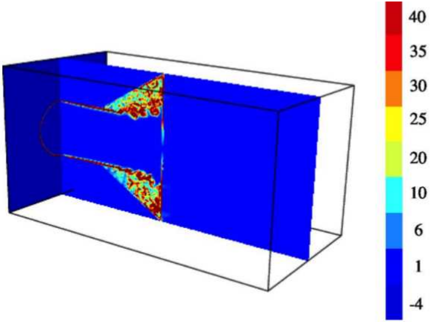


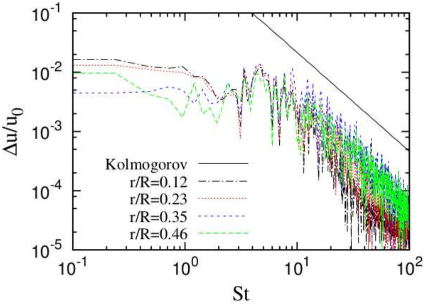

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- AMR allow us to reproduce the flow distribution inside a catalytic converter at much lower resolutions