Adaptive spectral wave forecasting

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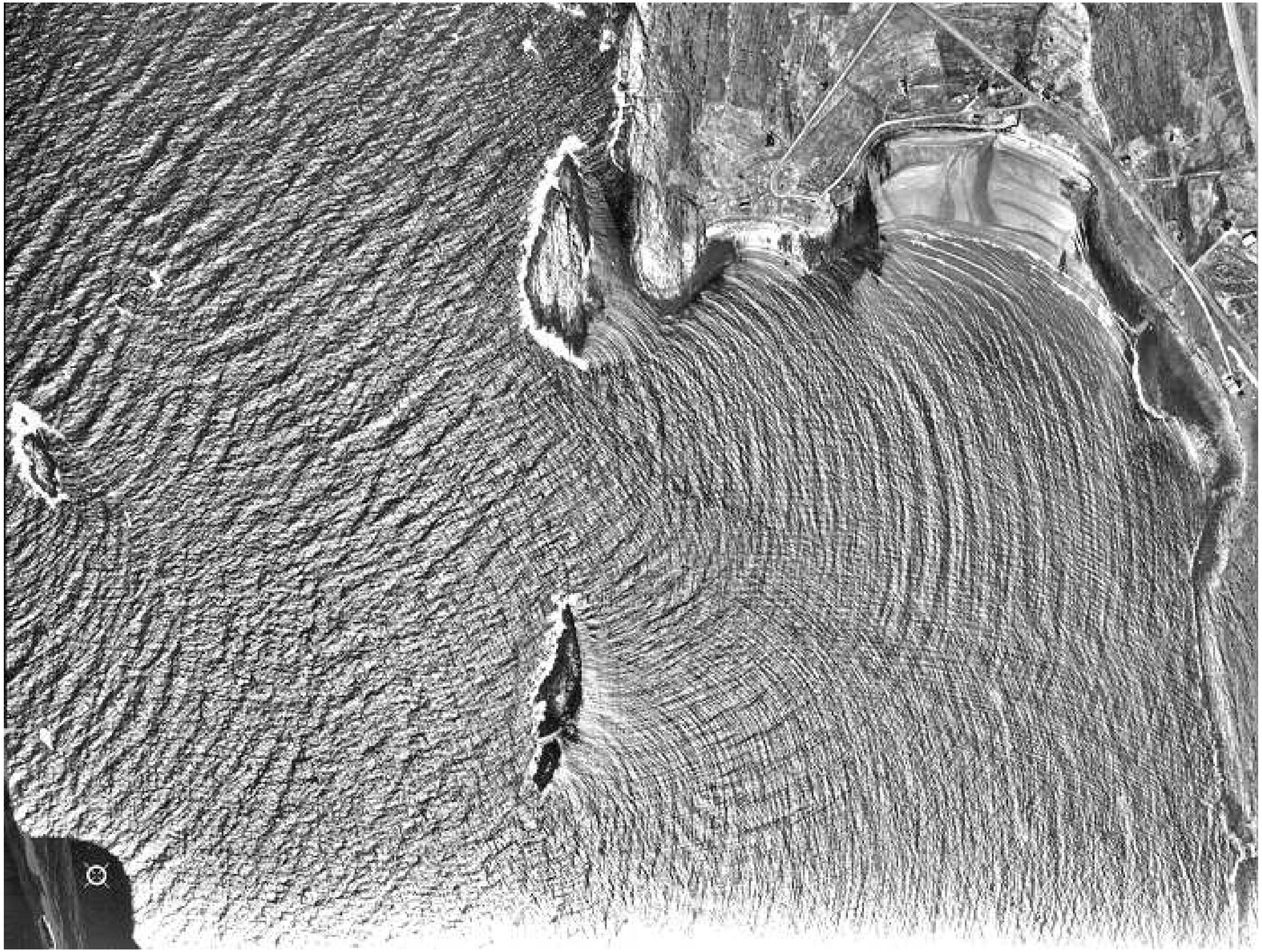
NIWA

NOAA/NCEP

Spectral description of a wave field



Statistical distribution of wavelengths and propagation directions (k, θ)

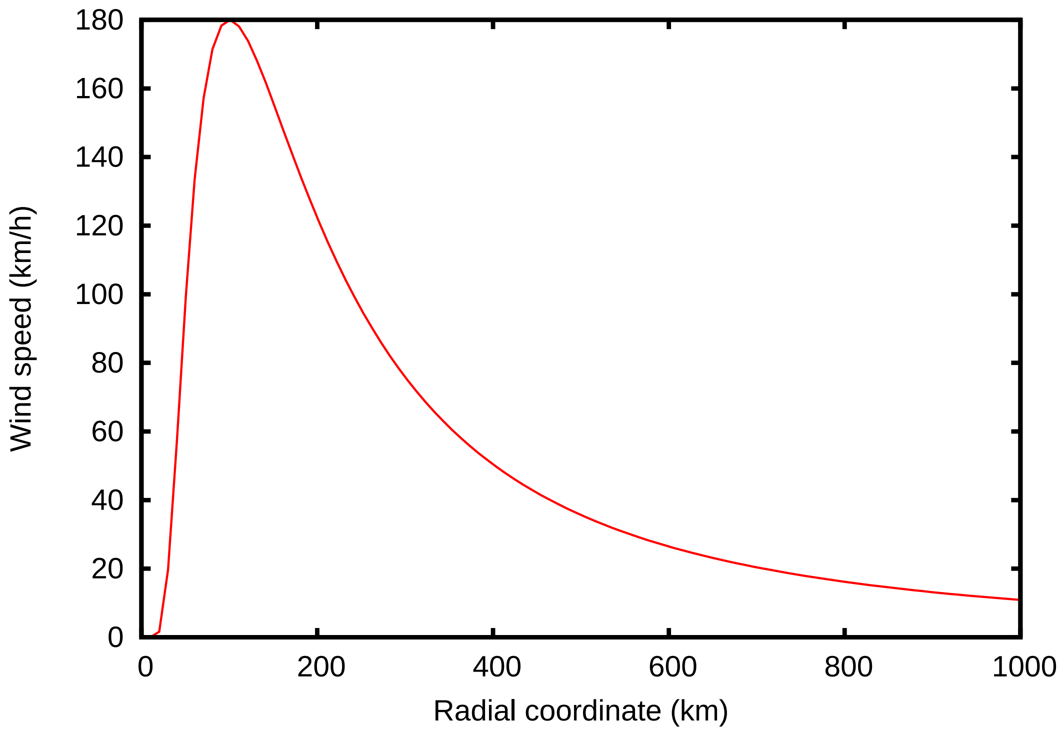
Spectral wave modelling

- Action density spectrum: $N(\mathbf{x}, k, \theta)$
- Based on Hasselman's (1970s) evolution equation for action density

$$\partial_t N + \nabla \cdot \mathbf{c_g} N + \partial_k \dot{k} N + \partial_\theta \dot{\theta} N = S/\sigma$$

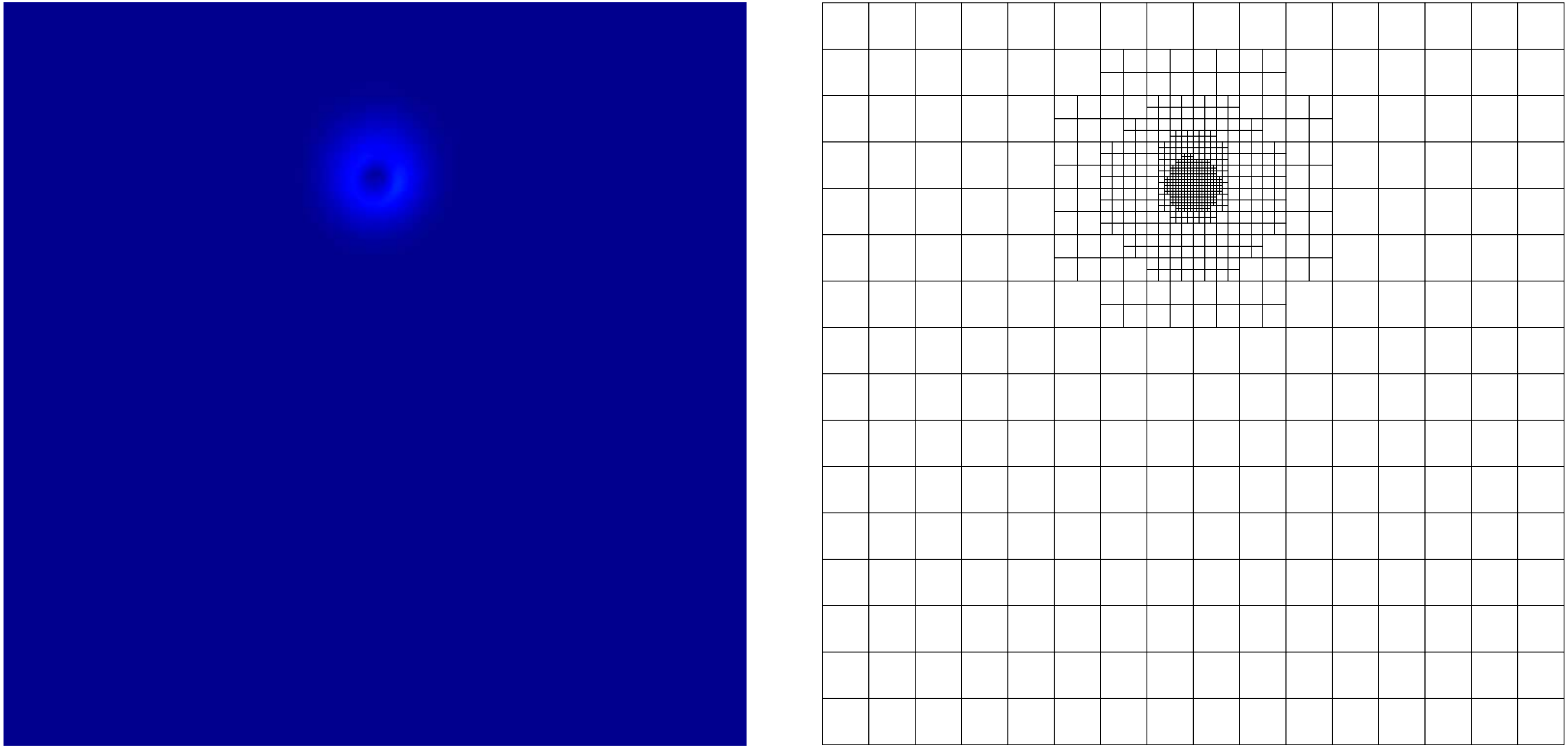
- Source terms S: wind-generated waves, non-linear wave/wave interactions, wave breaking etc...
- Very expensive: typically need to solve 25^2 advection problems per grid point
  $\Longrightarrow$ cost scales like $C\Delta_x^{-2}$ with C a large constant
- Advection in **x**-space using Gerris, other terms computed using WaveWatch III (Tolman, NOAA/NCEP)

Cyclone-generated wave field



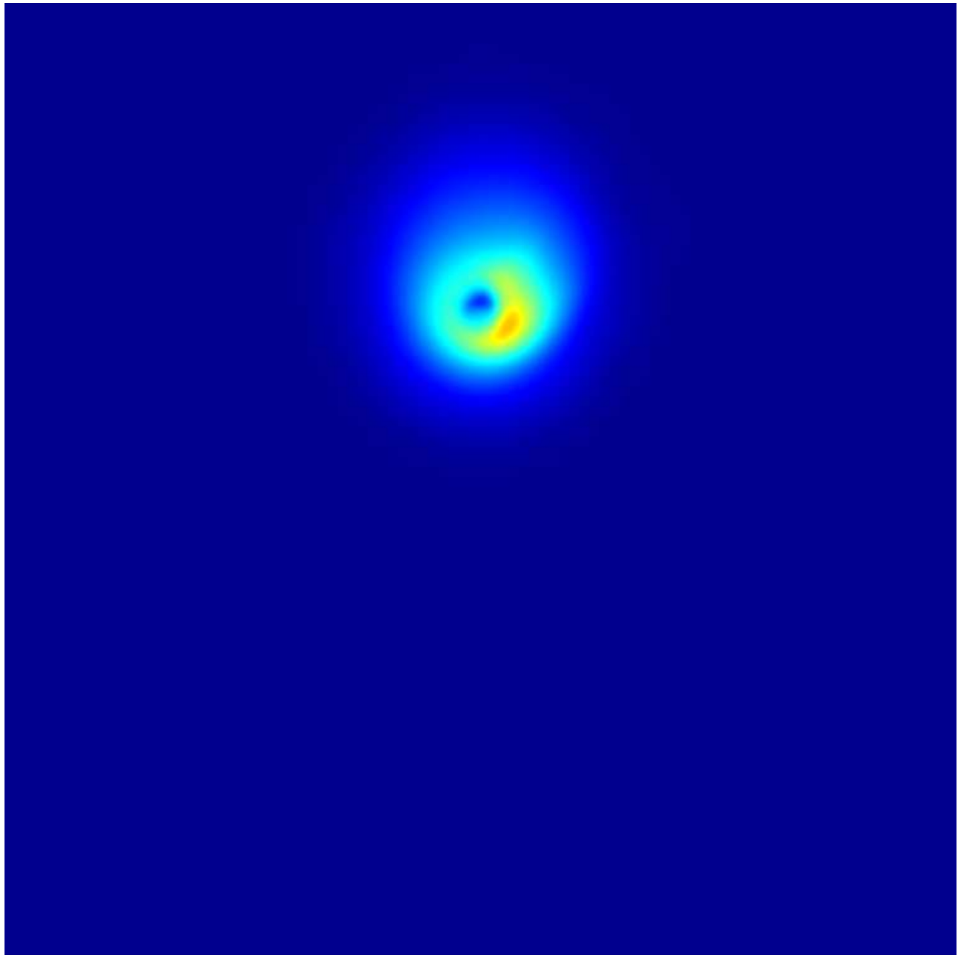
- Holland cyclone model
- Ramped linearly over 25 hours
- Moves south at 555 km/day
- Clockwise rotation

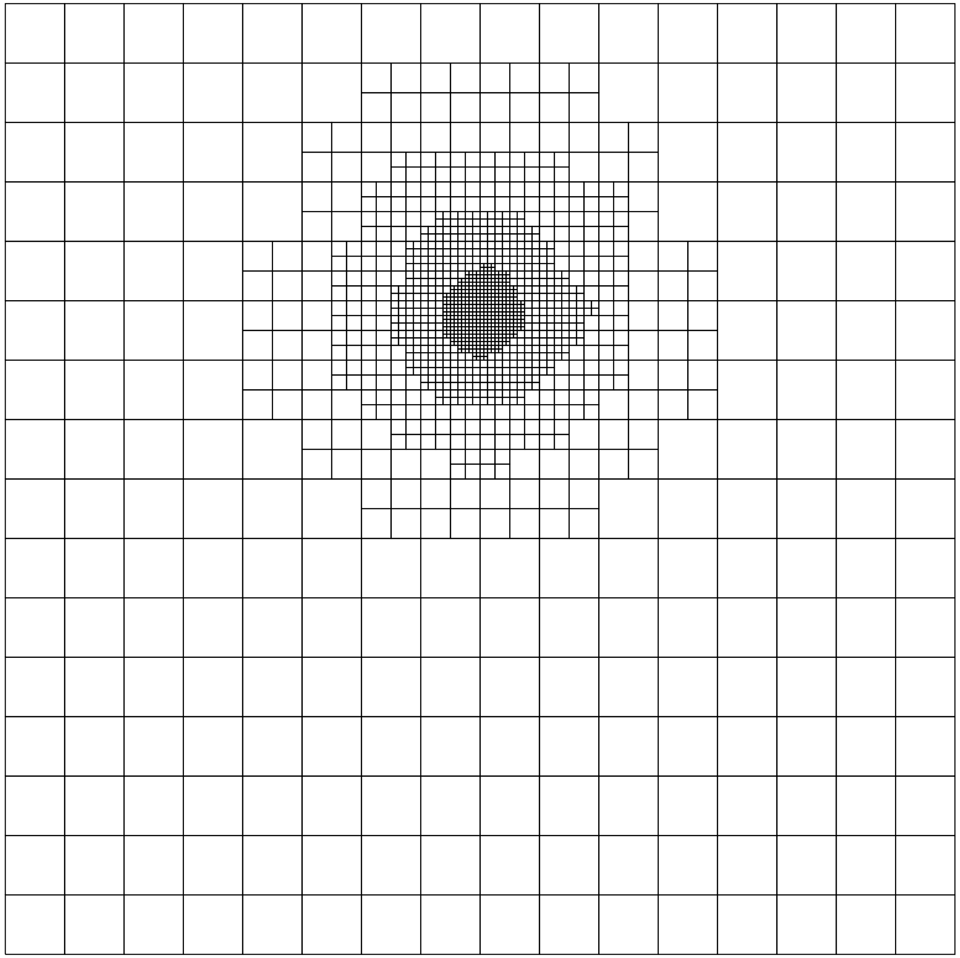
Evolution of the significant wave height



12 hours, max. wind 86 km/h, max. wave 3 m

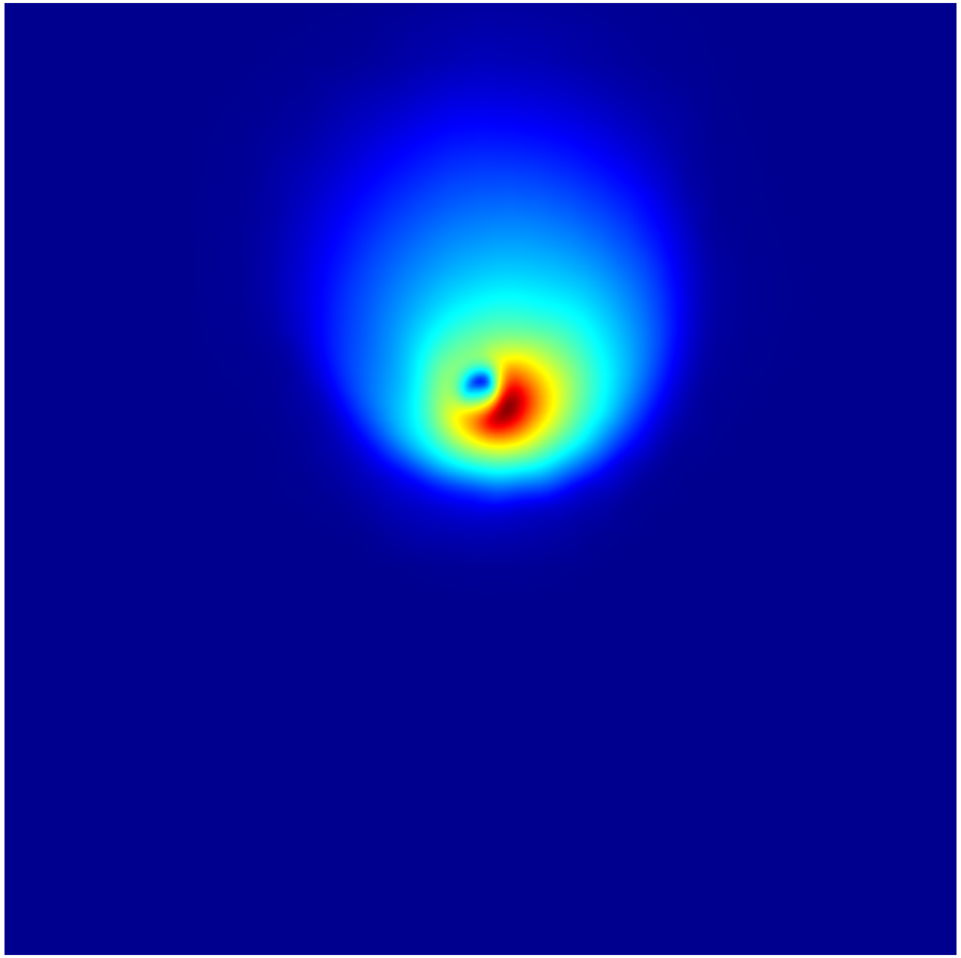
Evolution of the significant wave height

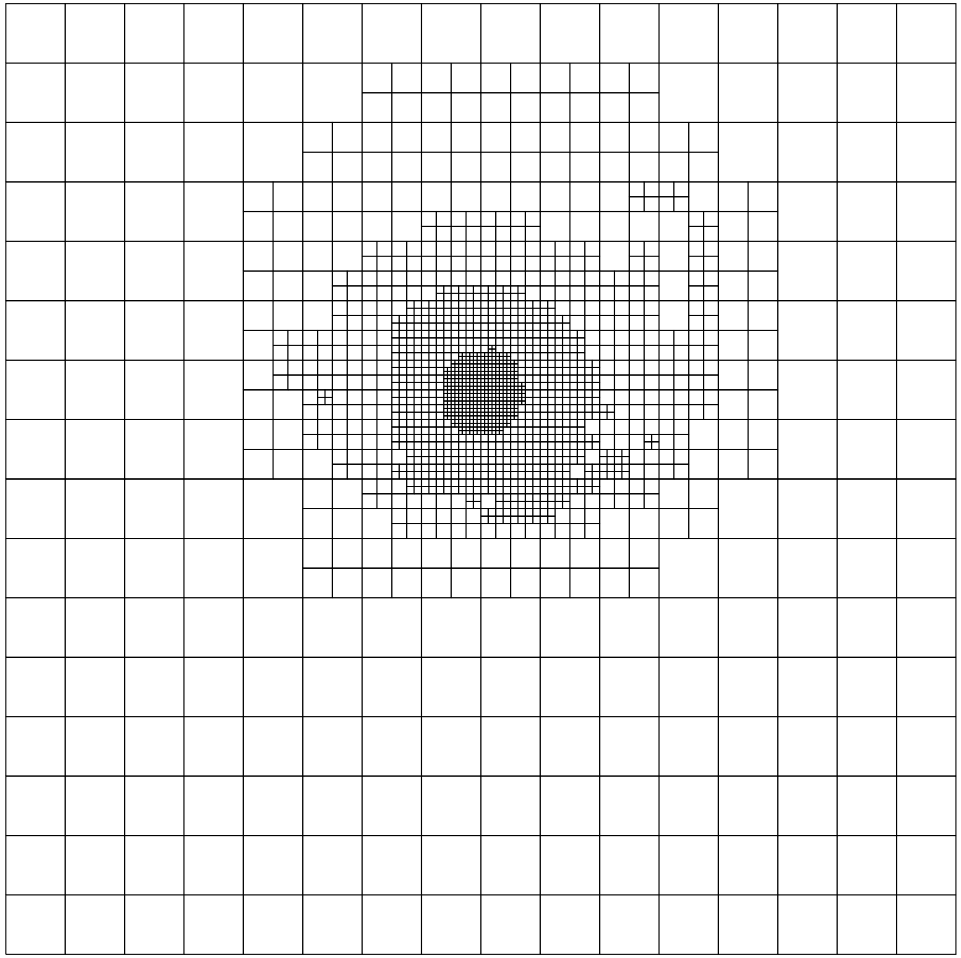




24 hours, max. wind 173 km/h, max. wave 14 m

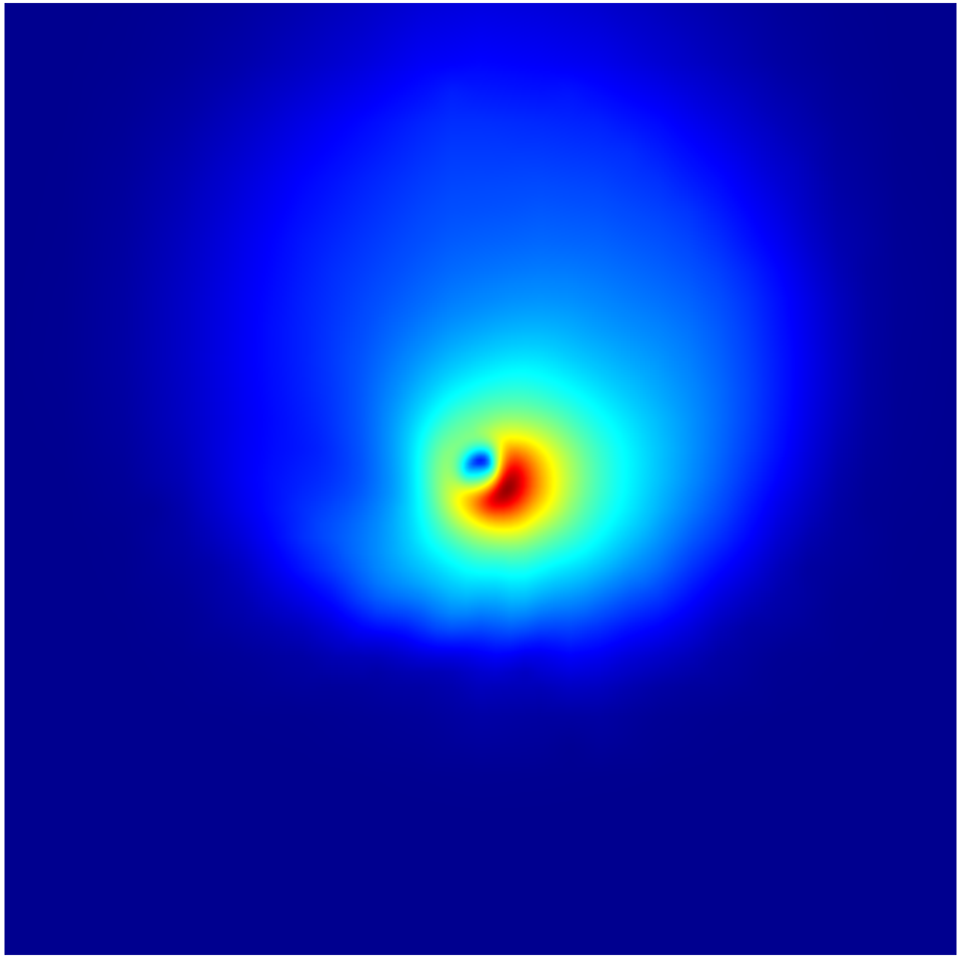
Evolution of the significant wave height

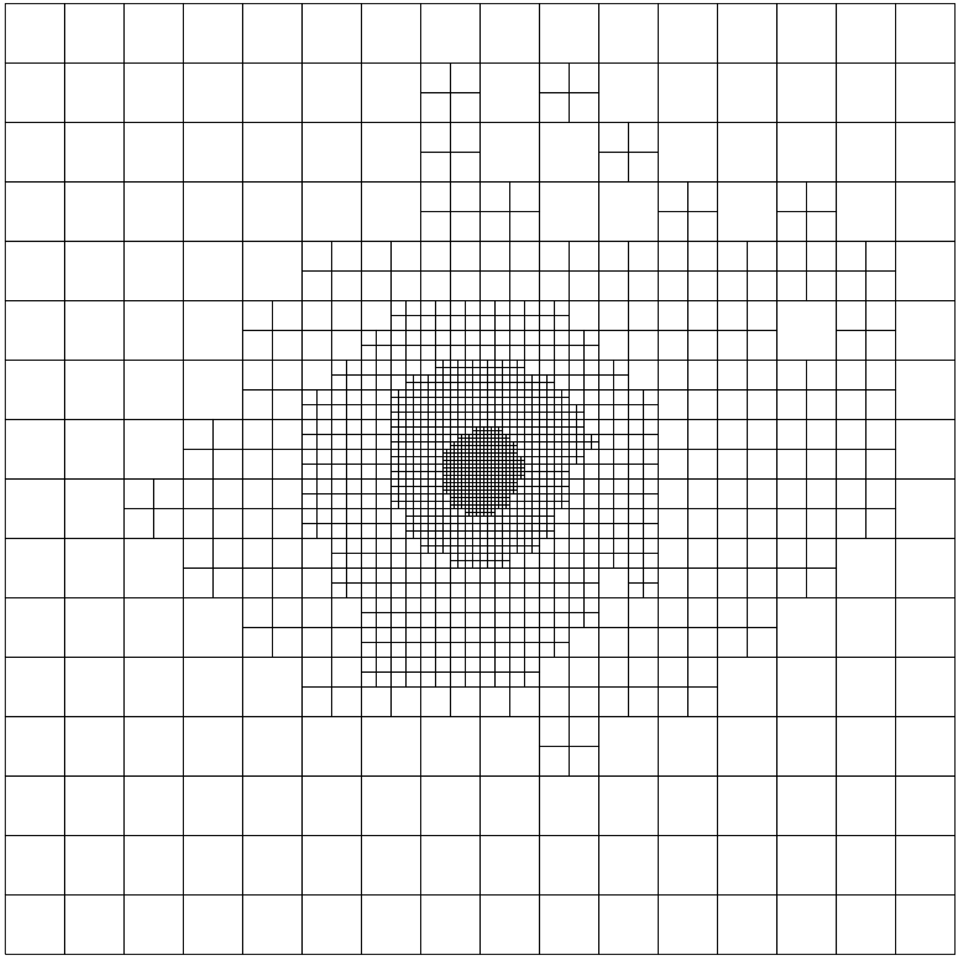




36 hours, max. wind 180 km/h, max. wave 20 m

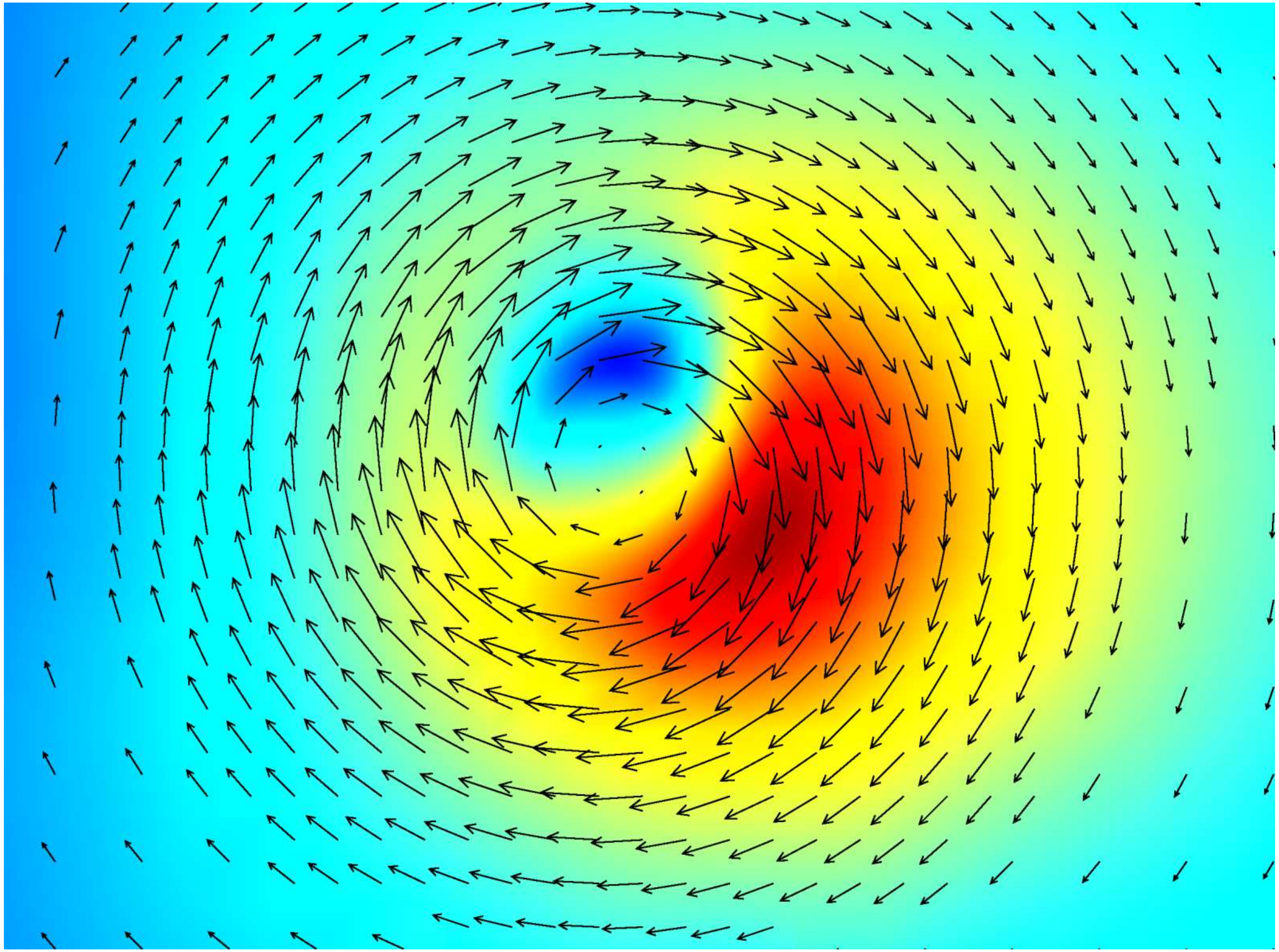
Evolution of the significant wave height



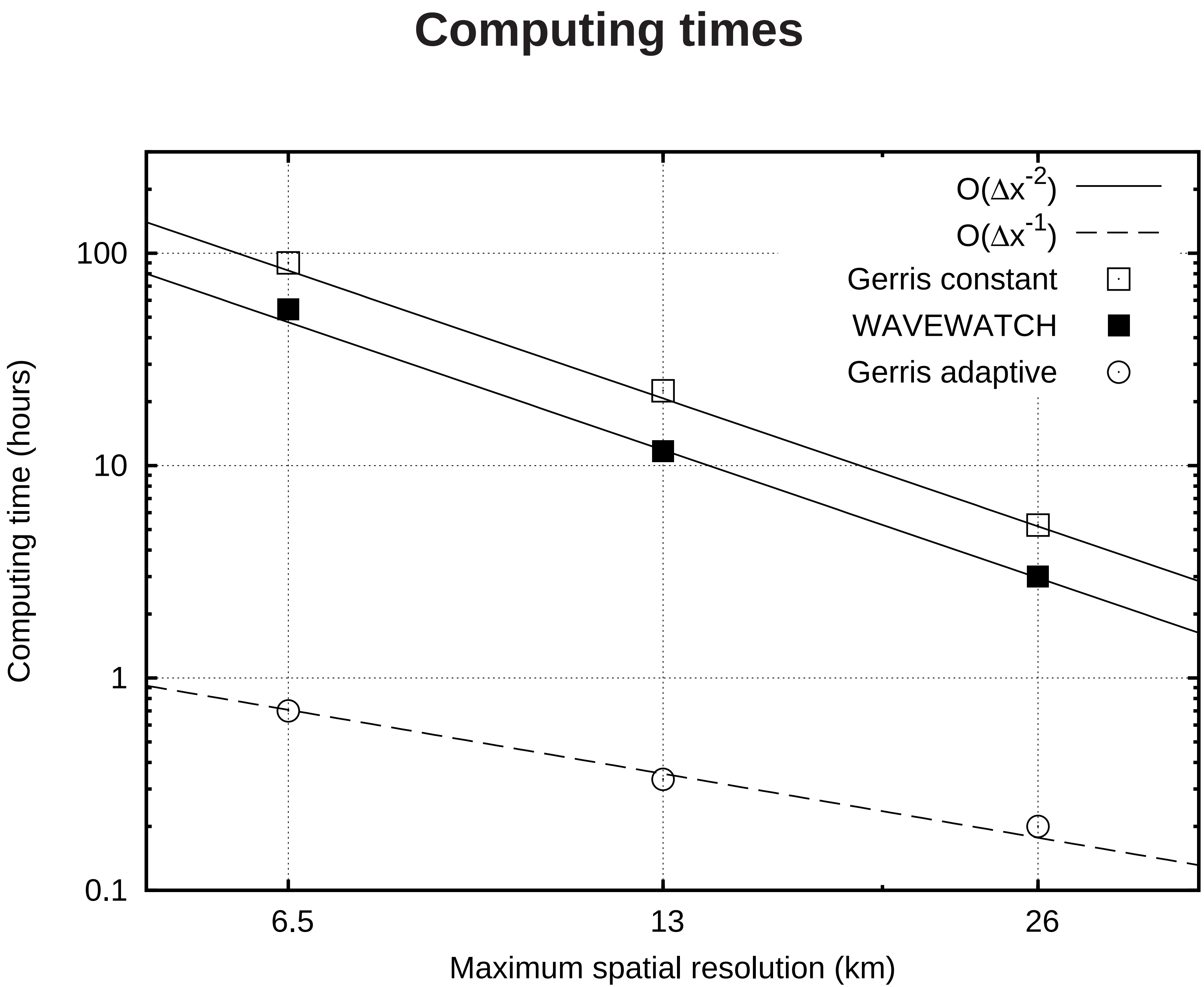


48 hours, max. wind 180 km/h, max. wave 20 m

In the eye of the cyclone

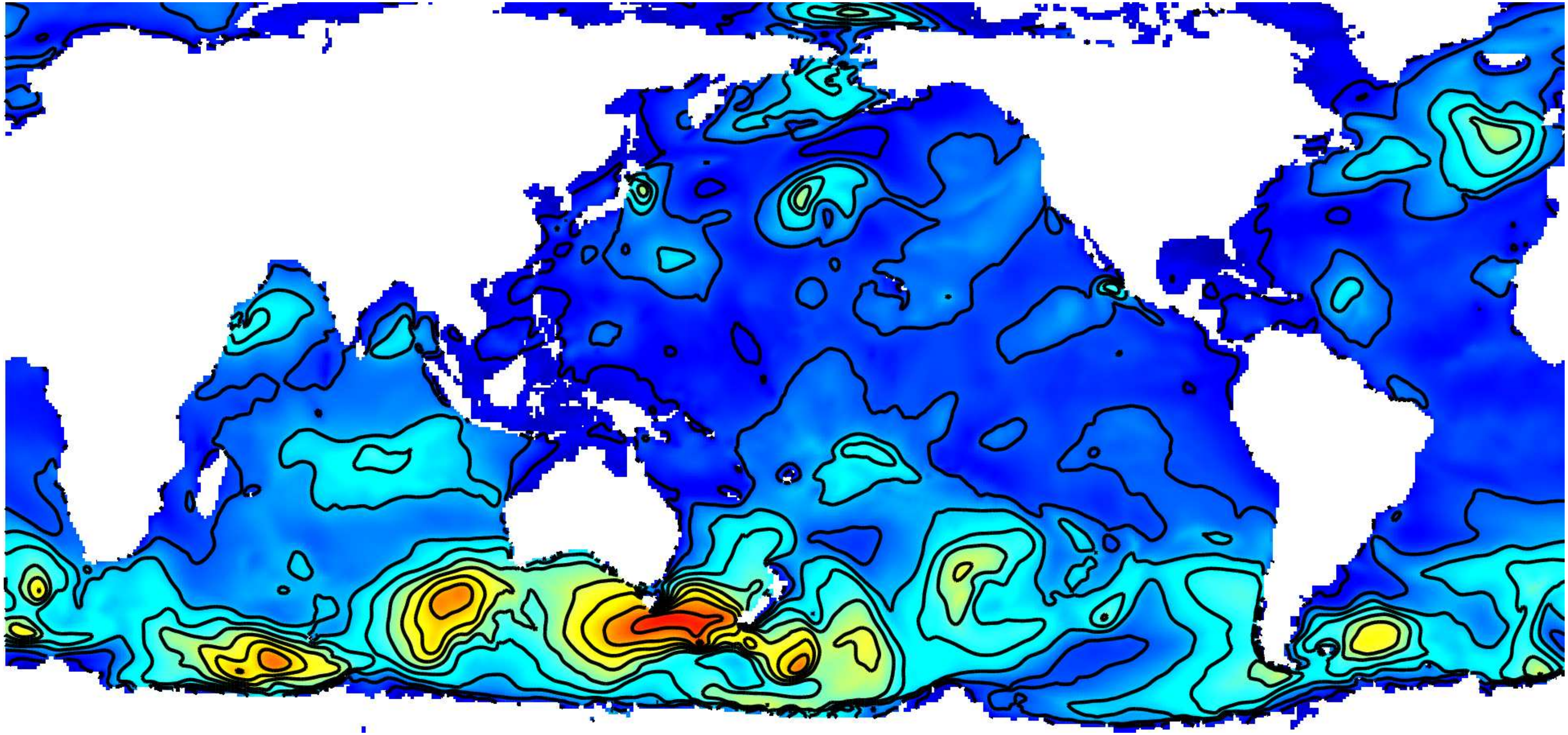


48 hours, max. wind 180 km/h, max. wave 20 m



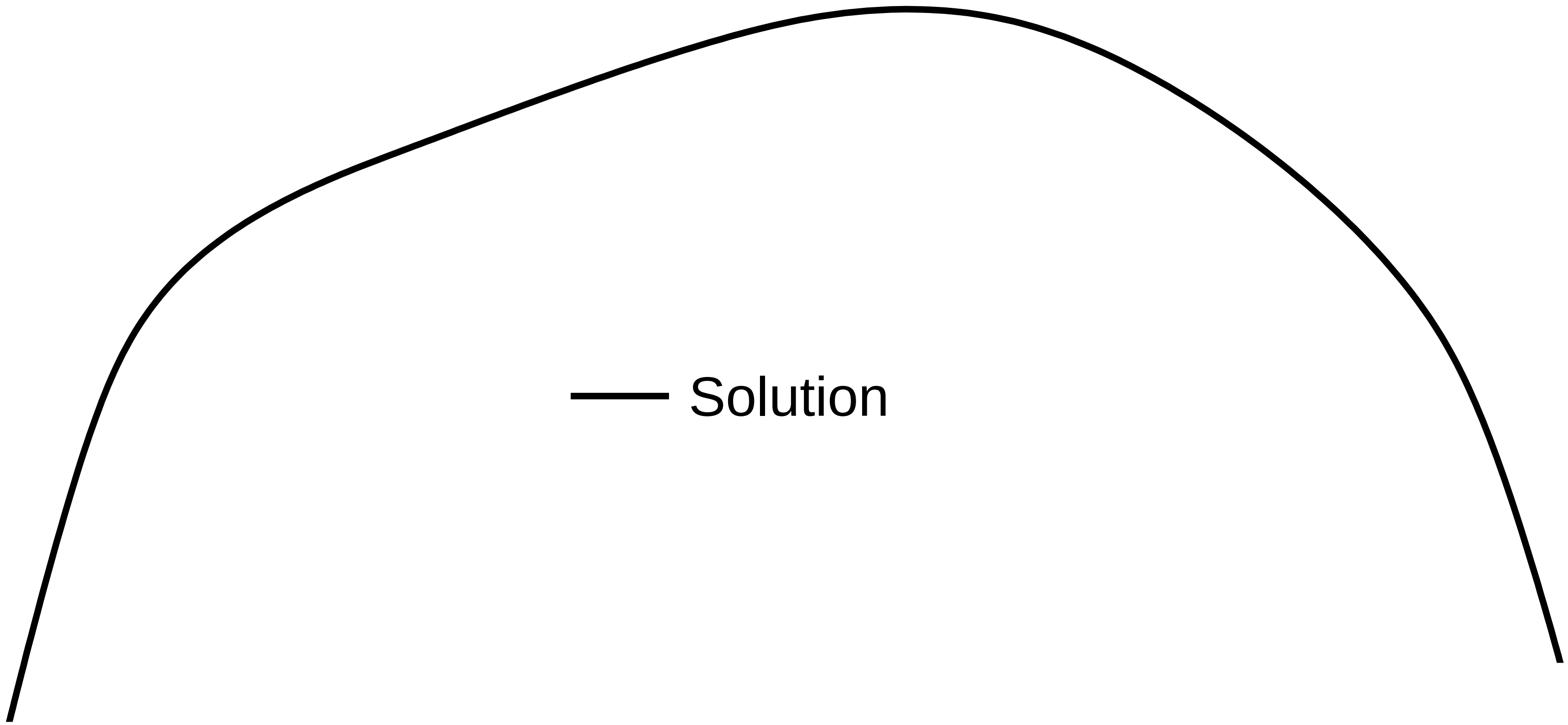
Computing times
O(Δx-2)
O(Δx-1)
Gerris constant
WAVEWATCH
Gerris adaptive
100
10
1
0.1
Computing time (hours)
6.5
13
26
Maximum spatial resolution (km)

Global forecast snapshot, NOAA/NCEP 1/9/2009

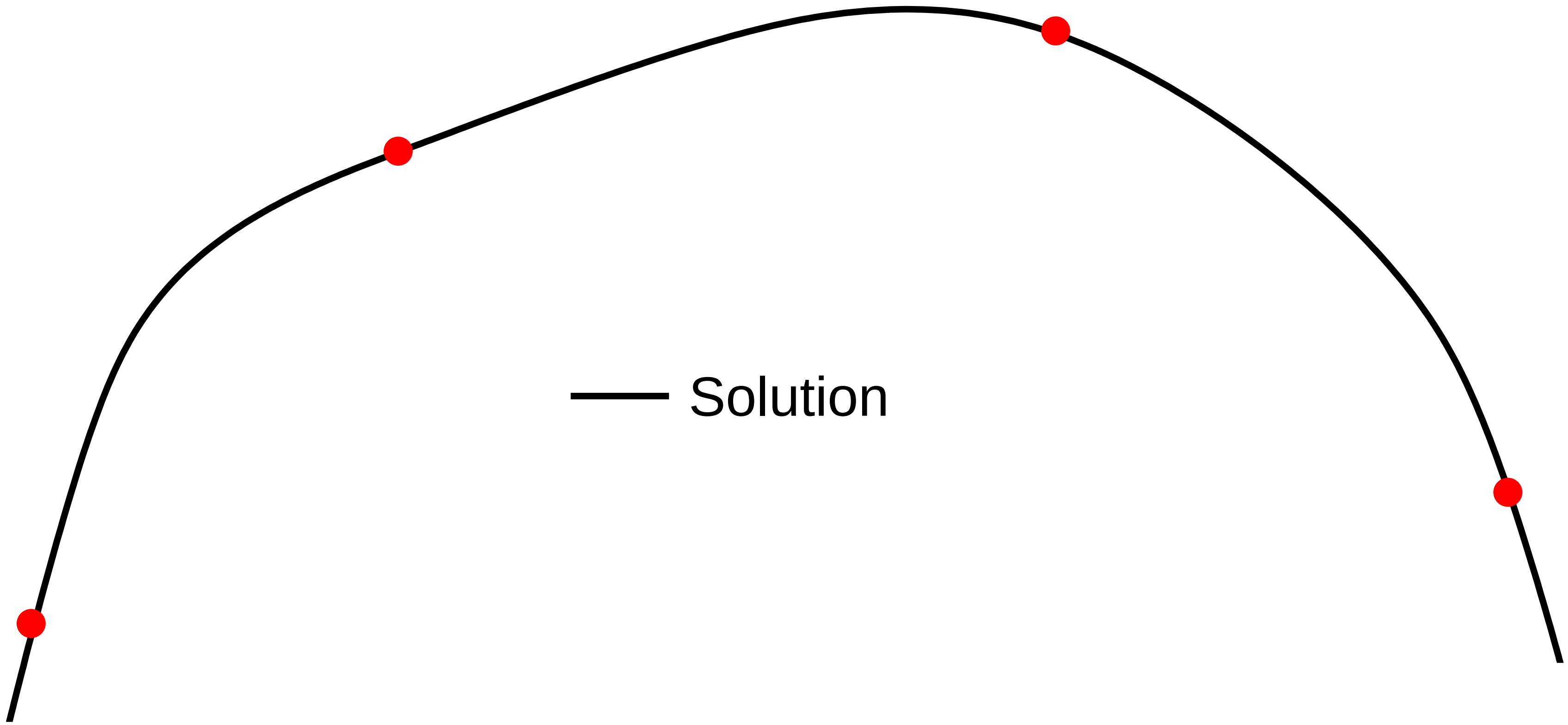


Significant wave height, dark red 8 metres, resolution 0.35 degrees, regular grid

Estimation of the “truncation error”



Estimation of the “truncation error”



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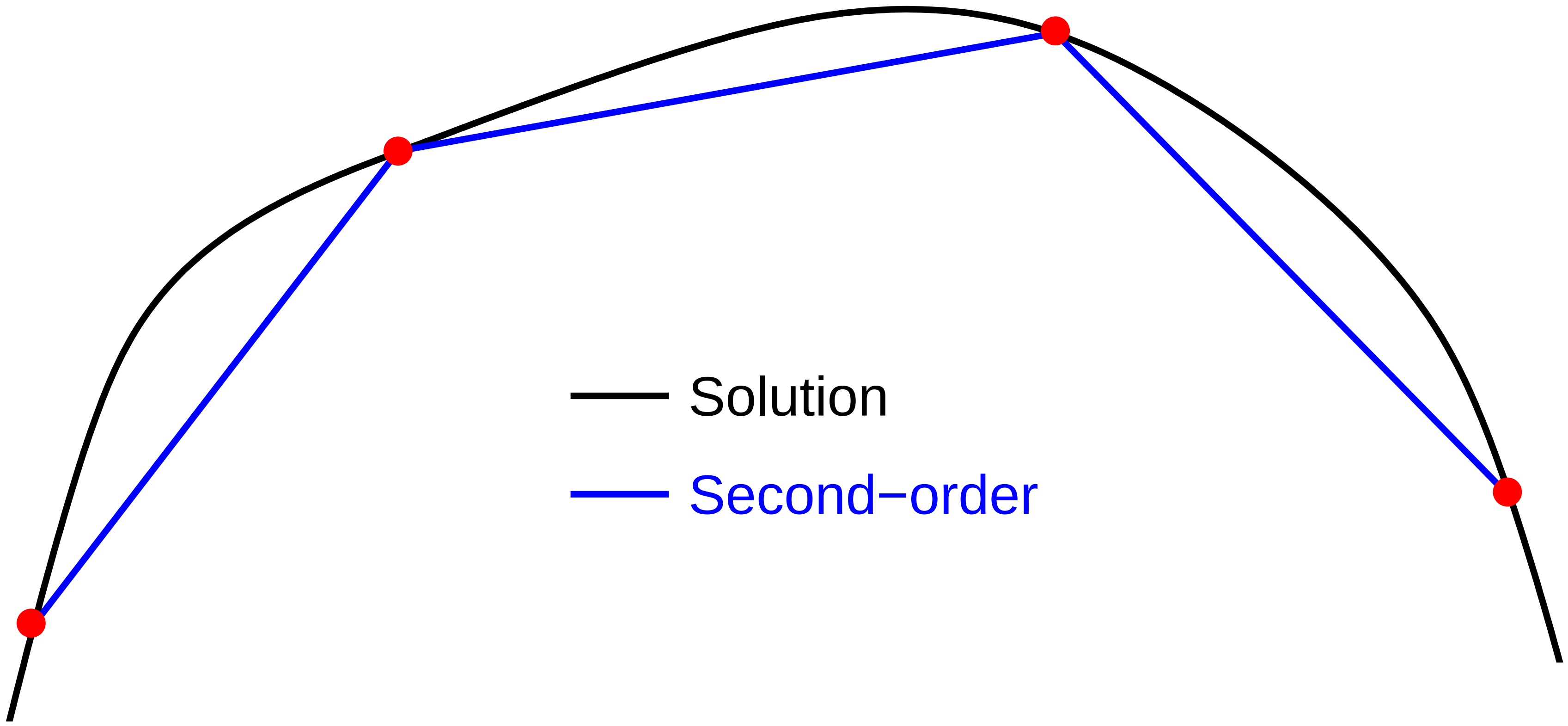

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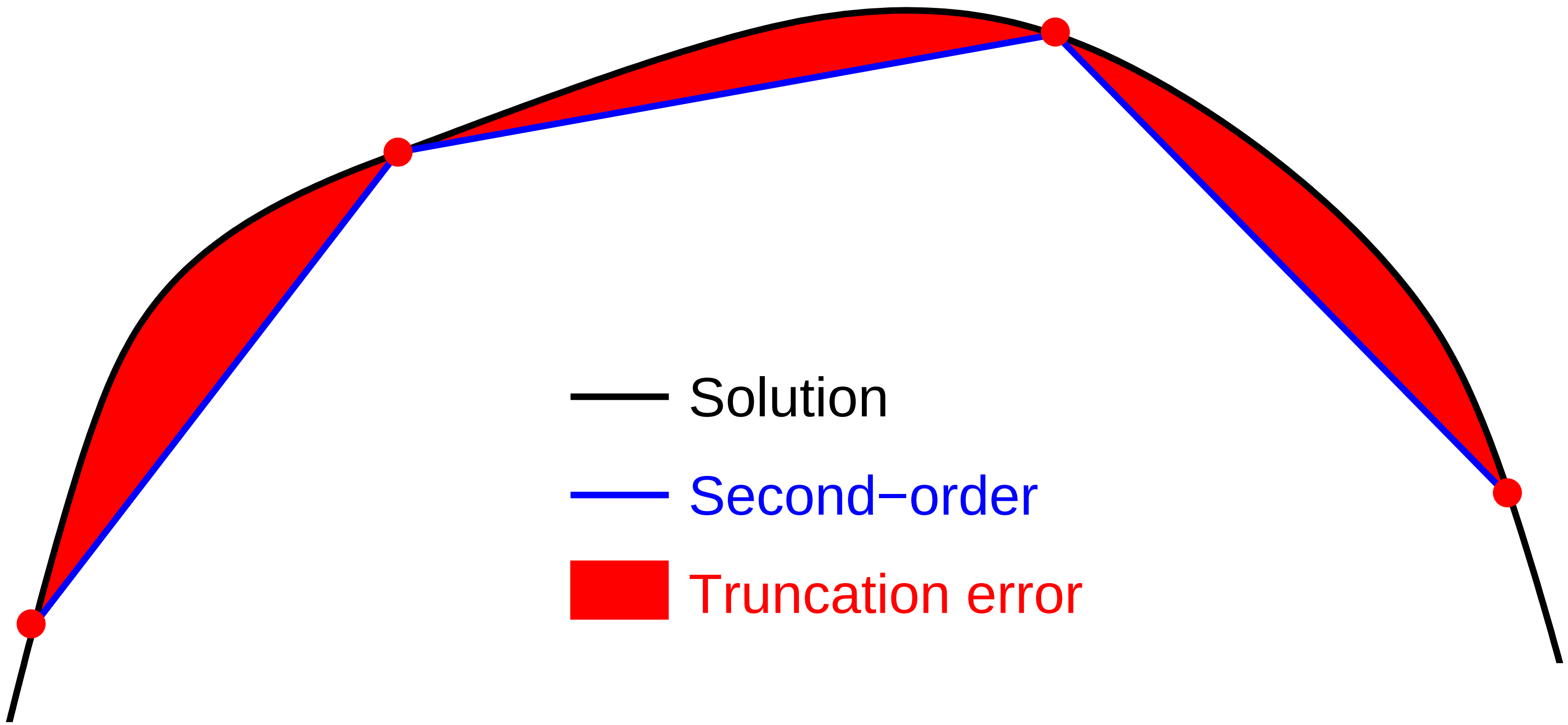
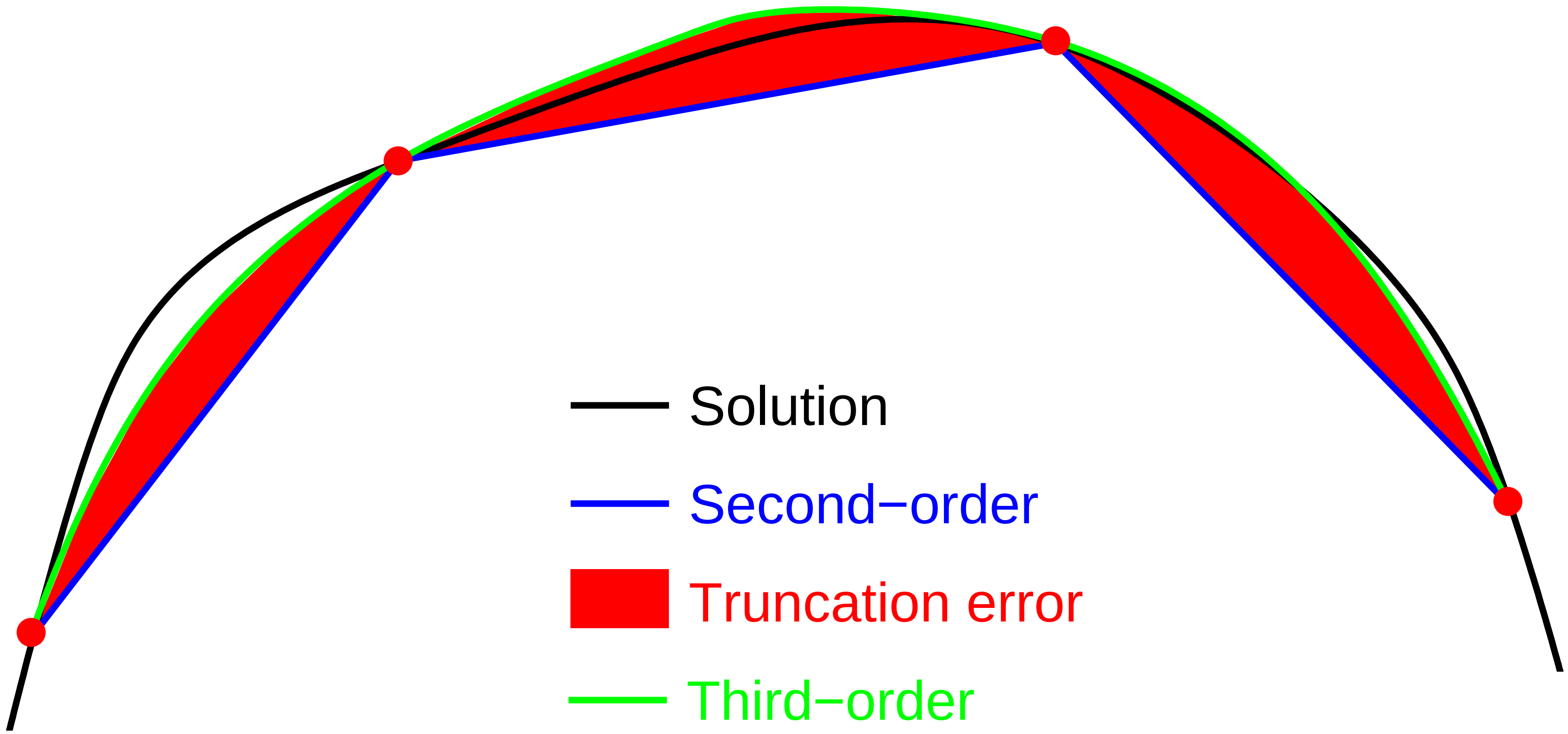
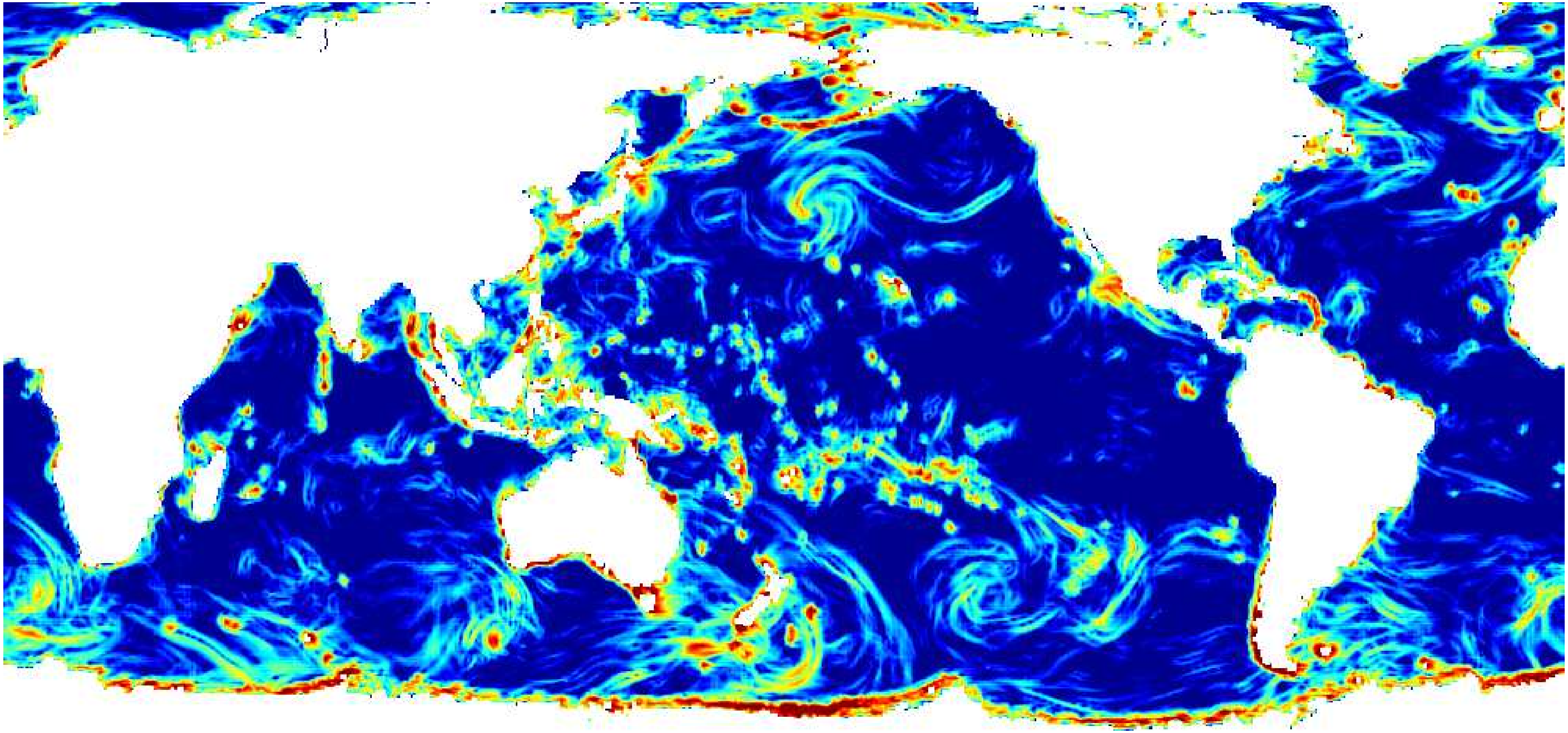


Estimation of the “truncation error”



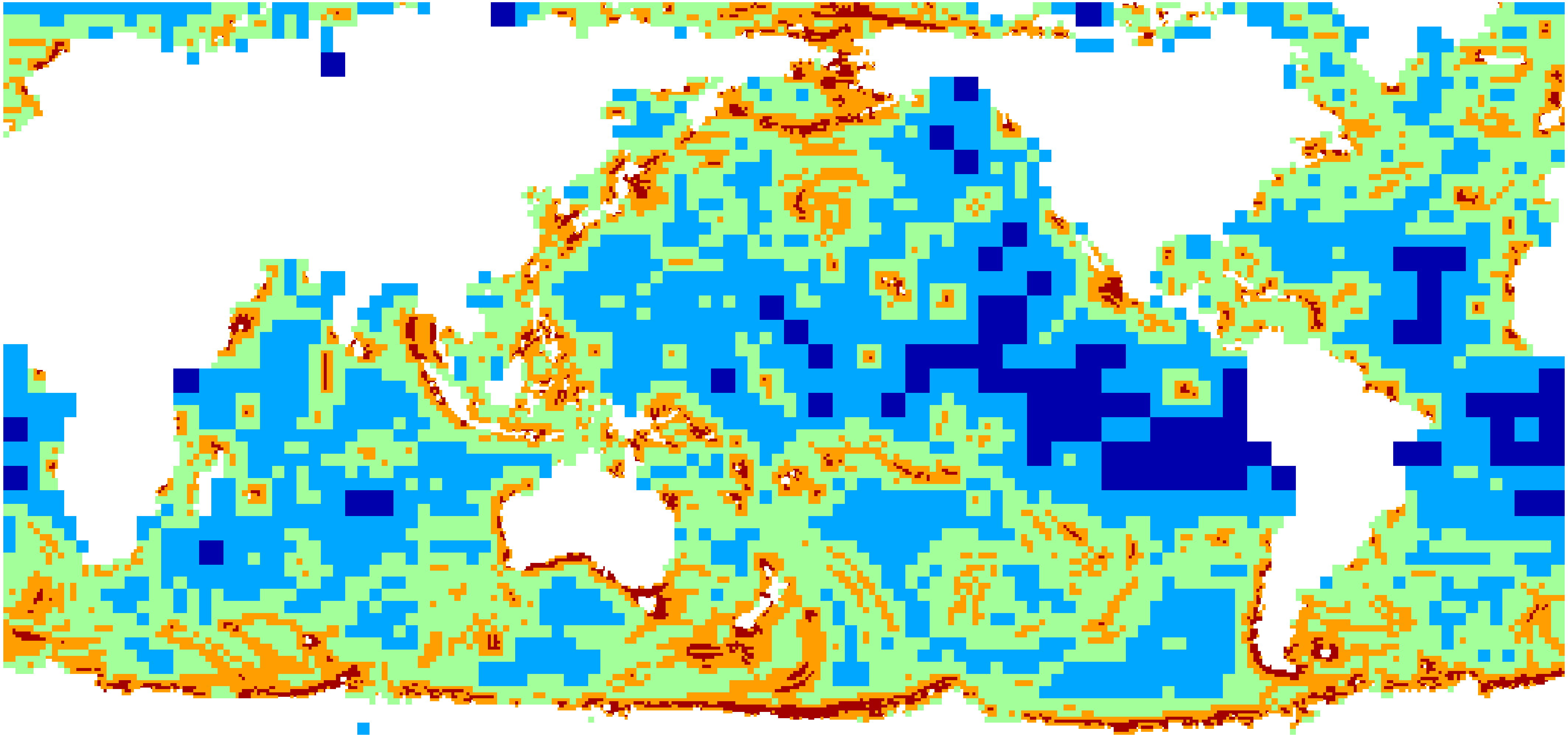
$$\approx \frac{\Delta_x^2}{8} ||\mathbf{F}''(\mathbf{x})||$$

Truncation error analysis



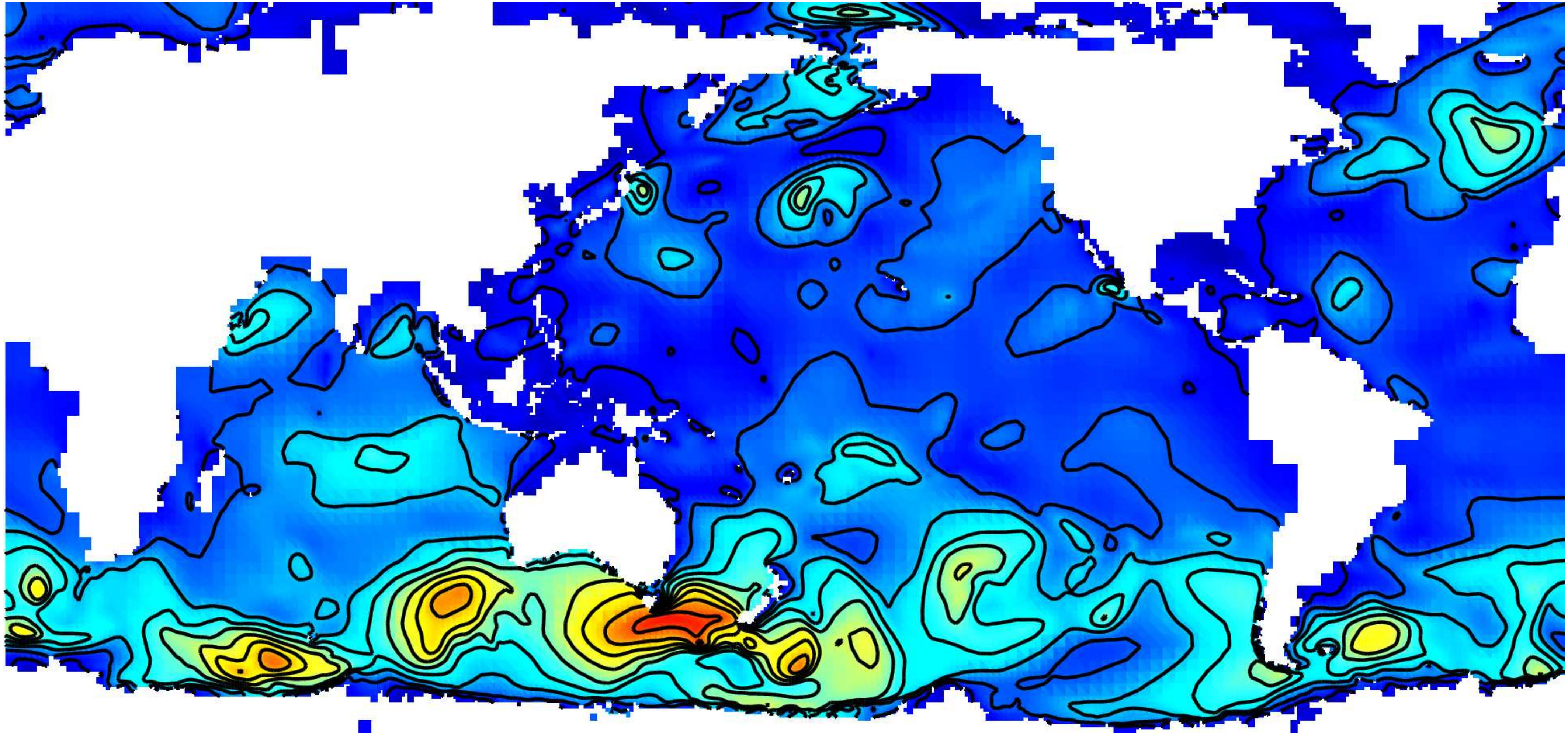
Dark blue < 1 mm, dark red > 5 cm

Adaptive mesh



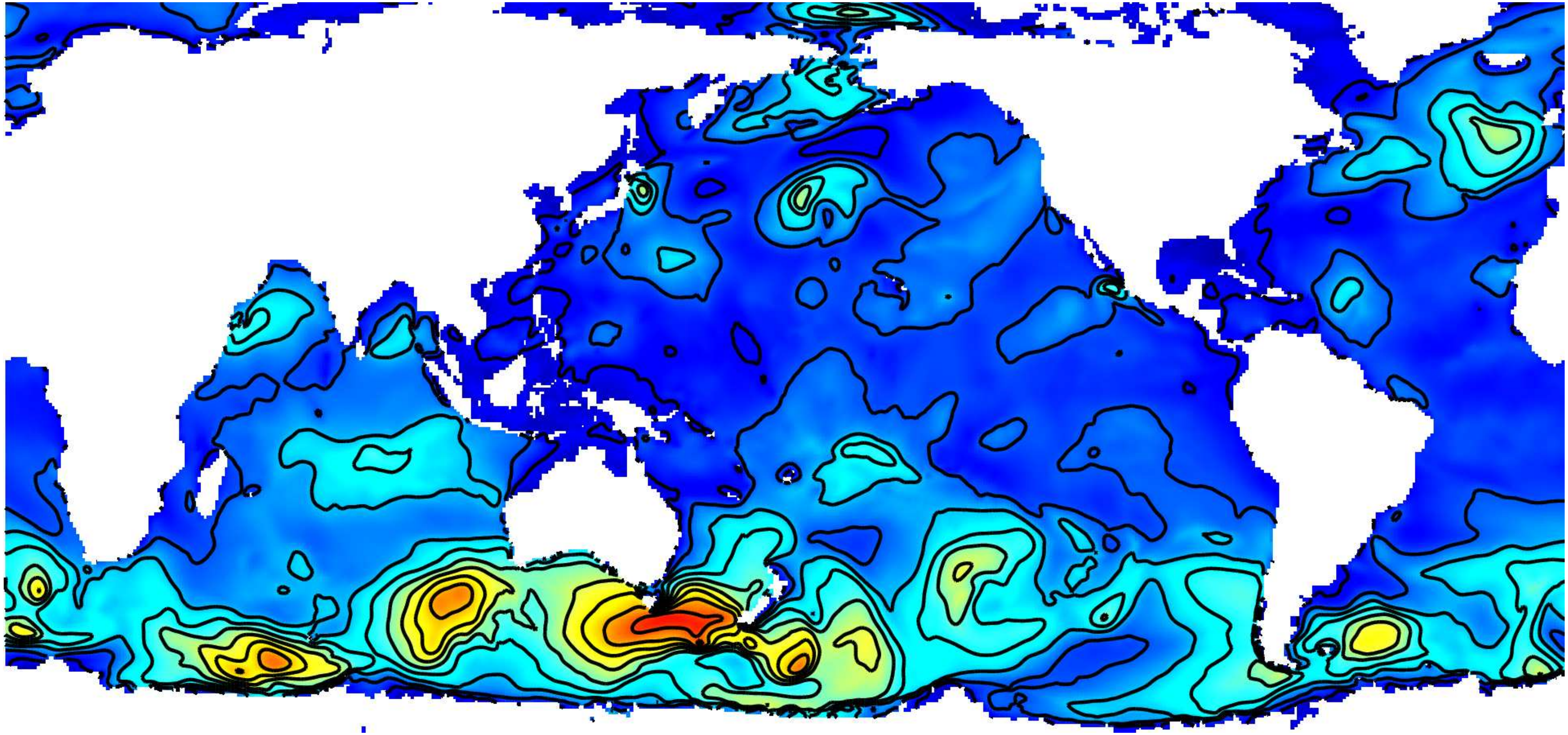
Truncation error < 5 cm, Dark red 0.35 degrees, dark blue 5.6 degrees

Adaptive resolution

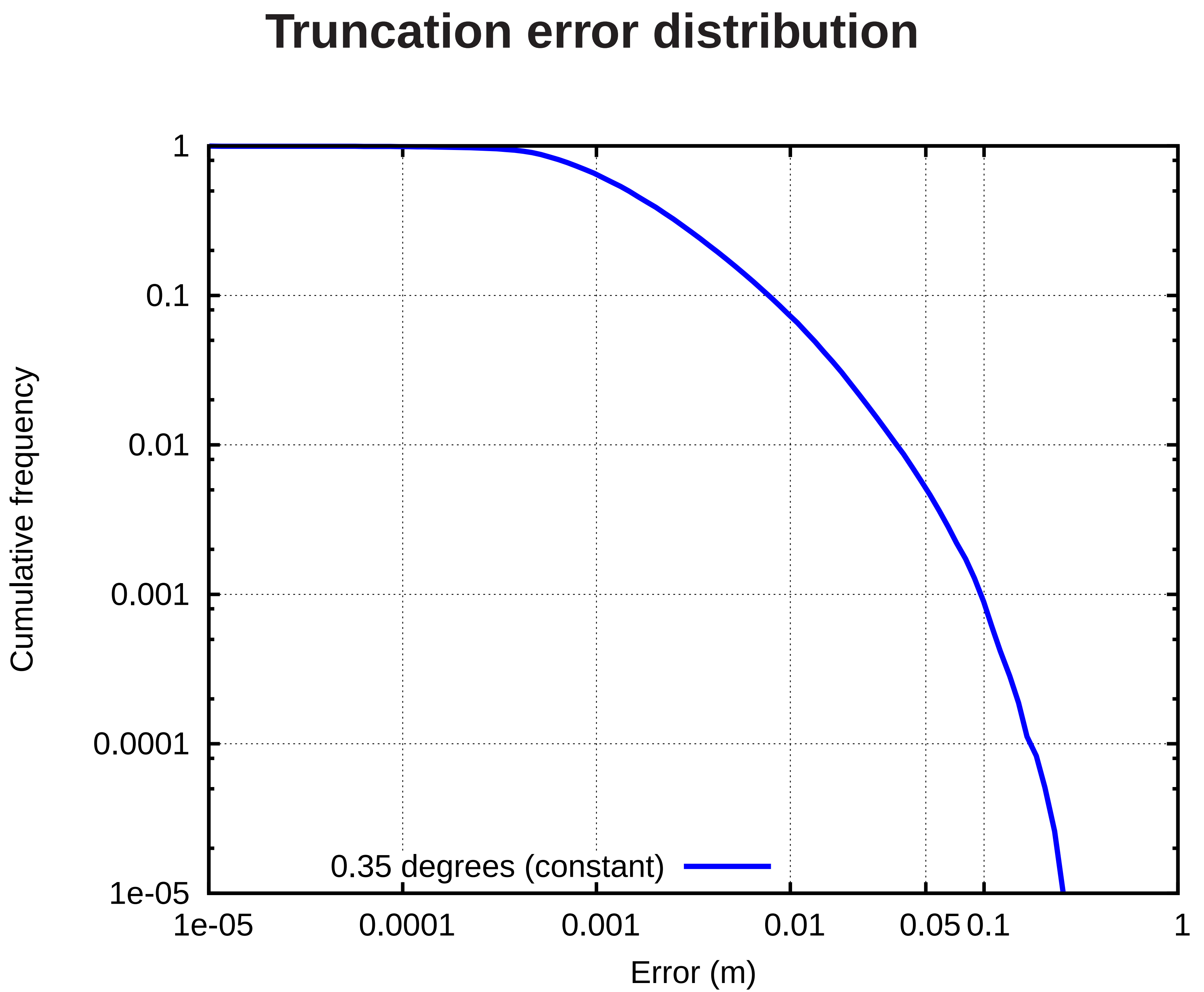


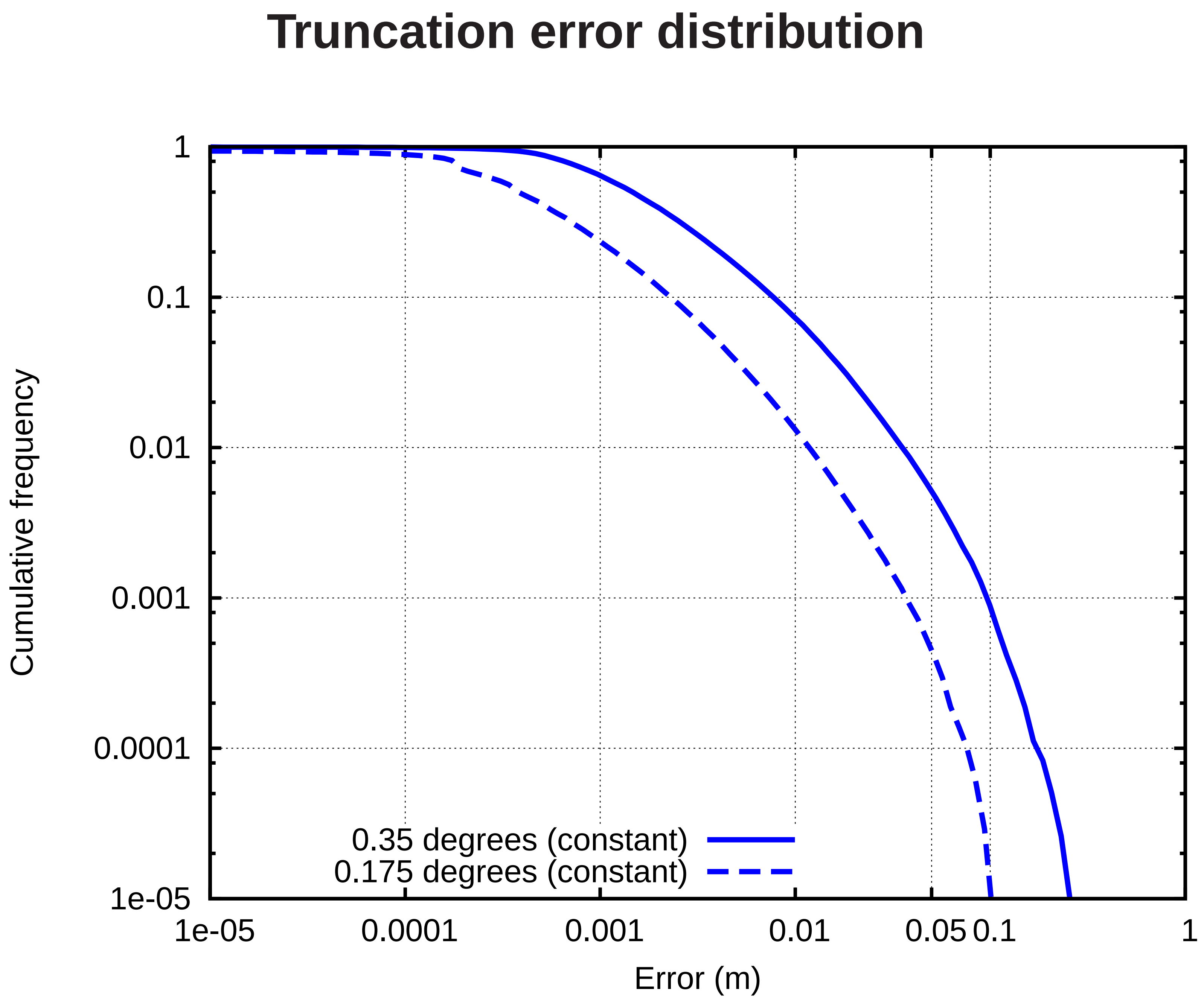
Truncation error < 5 cm, 0.35 degrees, $\approx$ 30,000 grid points

Constant resolution

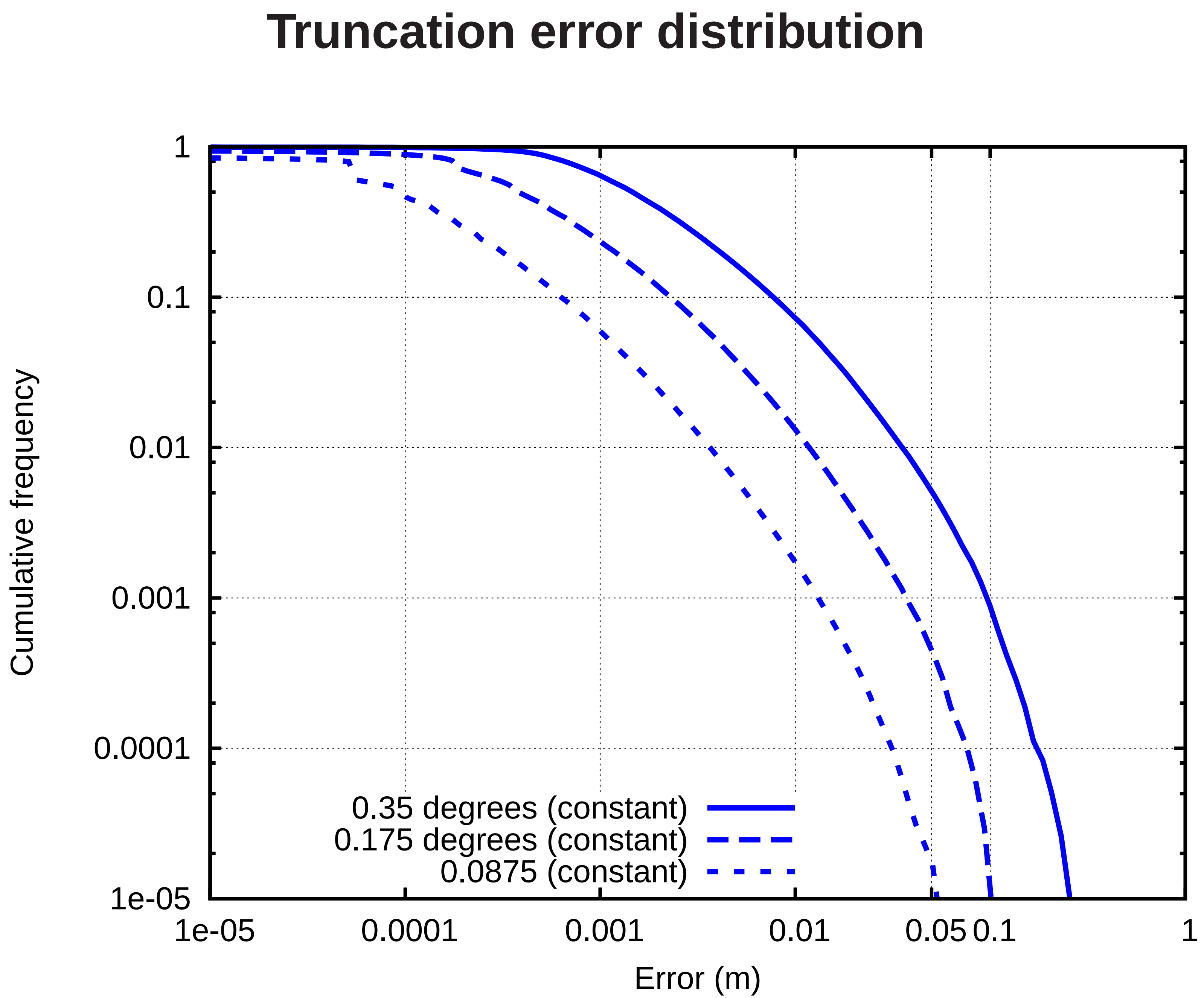


0.35 degrees, $\approx$ 300,000 grid points

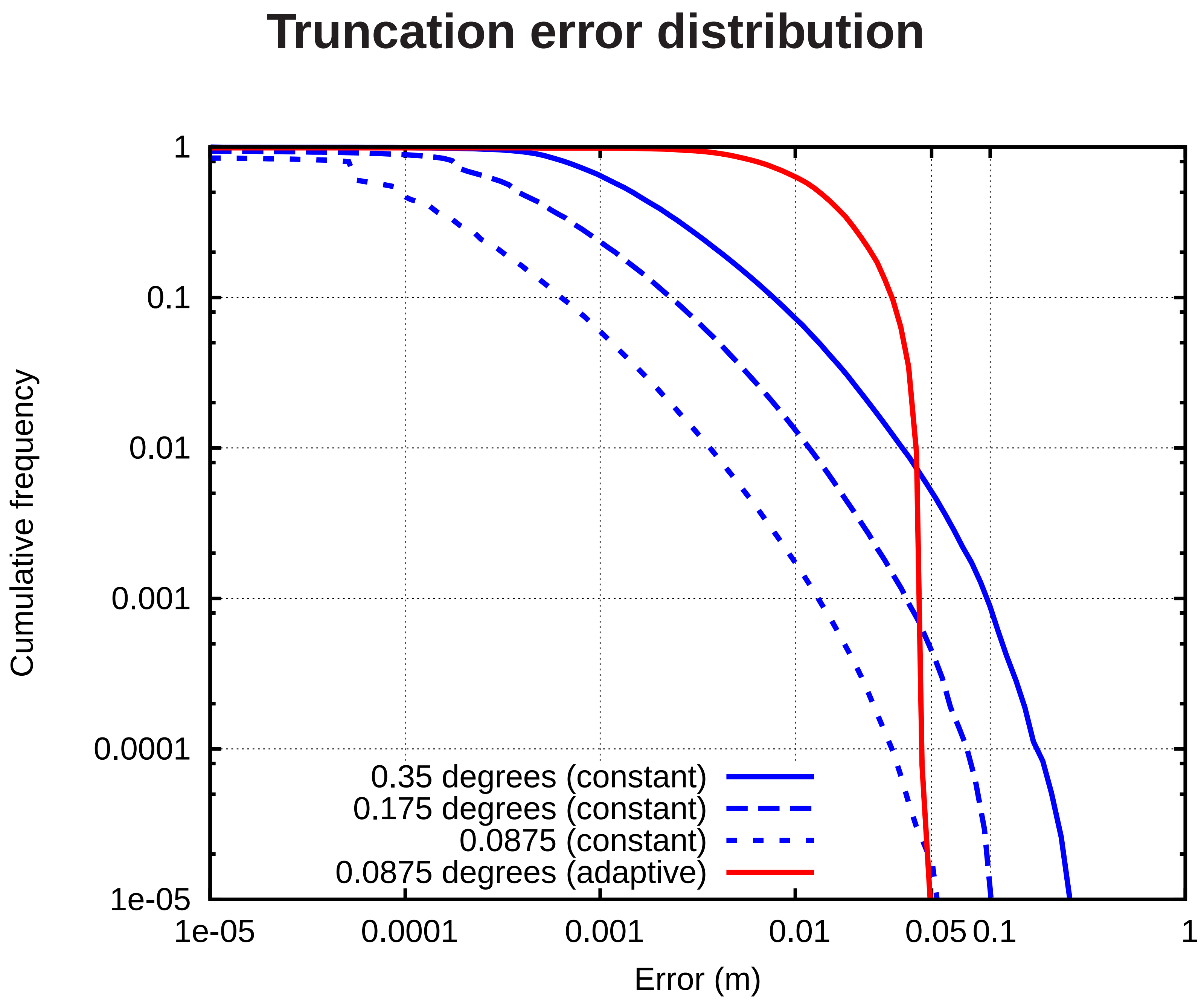

Truncation error distribution
1
0.1
0.01
0.001
0.0001
1e-05
Cumulative frequency
1e-05
0.0001
0.001
0.01
0.05
0.1
1
Error (m)
0.35 degrees (constant)



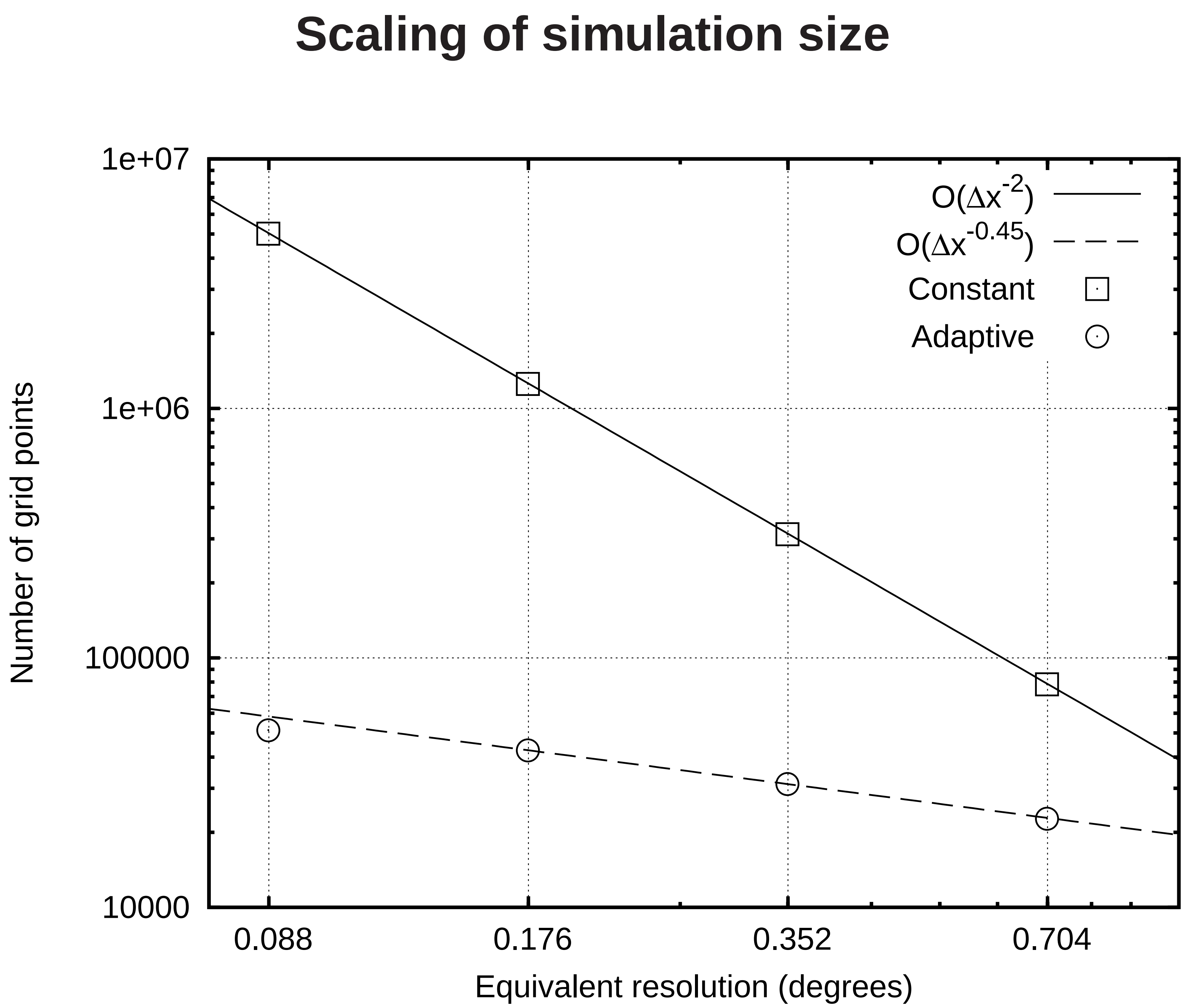
Truncation error distribution
Cumulative frequency
1
0.1
0.01
0.001
0.0001
1e-05
1e-05
0.0001
0.001
0.01
0.05
0.1
1
Error (m)
0.35 degrees (constant)
0.175 degrees (constant)



Truncation error distribution
Cumulative frequency
1
0.1
0.01
0.001
0.0001
1e-05
1e-05
0.0001
0.001
0.01
0.05
0.1
1
Error (m)
0.35 degrees (constant)
0.175 degrees (constant)
0.0875 (constant)



Truncation error distribution
Cumulative frequency
1
0.1
0.01
0.001
0.0001
1e-05
1e-05
0.0001
0.001
0.01
0.05
0.1
1
Error (m)
0.35 degrees (constant)
0.175 degrees (constant)
0.0875 (constant)
0.0875 degrees (adaptive)


Scaling of simulation size
Number of grid points
Equivalent resolution (degrees)
1e+07
1e+06
100000
10000
0.088
0.176
0.352
0.704
O(Δx^-2)
O(Δx^-0.45)
Constant
Adaptive

Conclusions and future work

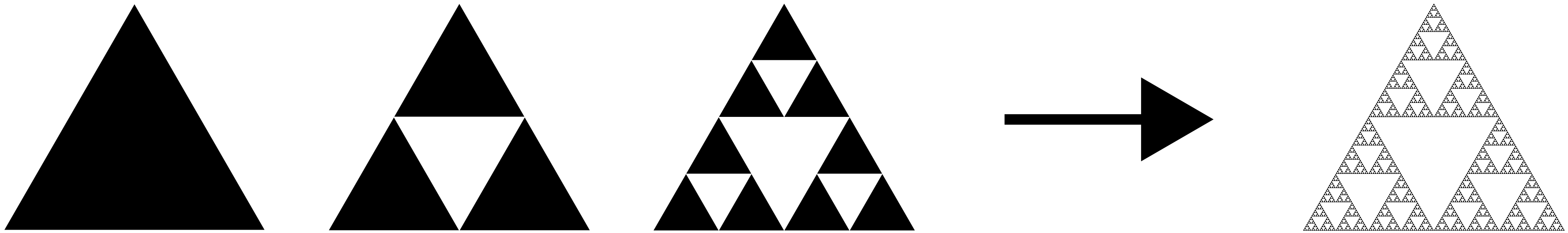
- Adaptivity changes the scaling of computing costs: $C\Delta_x^{-p}$, p is now (much) smaller than the number of dimensions
- This leads to orders-of-magnitude savings $\Longrightarrow$ new possibilities
- Benefits of adaptivity can be assessed beforehand using the scaling analysis we presented
- Popinet, Gorman, Rickard and Tolman, *Ocean Modelling*, **34**, 2010

Work in progress

- Application to cyclone-generated waves/hazard forecasting in the Pacific region
- Extension to waves in shallow water

Because it is an approximation of the *fractal dimension* of the solution

Classical example: the Sierpinski triangle



has a fractal (Minkowski or “box-counting” or “information”) dimension of ≈ 1.6.

In other words, the cost of describing such an object using quadtrees would scale as $N^{1.6}$ not N^2.