Lake discussions

November 16, 2009

At the free surface, the shear stress is evaluated by the wind velocity:

$$\tau_s = \rho_a C_D u_w^2 \tag{1}$$

with ρ_a density of air, C_D = wind drag coefficient, and eventually, u_w = wind velocity at 10 m from the surface. By definition one introduces u_{*s} the free surface shear velocity

$$u_{*s}^2 \equiv \tau_s / \rho \tag{2}$$

with ρ density of water

Because of a shear linear in z and a constant diffusion constant, we must assume a parabolic velocity profile

$$u(z) = u_{*s}[A(\frac{z}{d})^2 + B\frac{z}{d} + C]. \tag{3}$$

To determine the constants A, B and C, we set the following conditions

1. At the free surface, the shear stress is evaluated by the wind velocity:

$$\rho D_{effective} \frac{\partial u}{\partial z}\bigg|_{z=d} = \tau_s, \tag{4}$$

2. At the bottom, the velocity is equal to zero:

$$u(0) = 0. \tag{5}$$

3. The depth-averaged velocity equals zero:

$$\frac{1}{d}\int_0^d u(z)dz = 0 \tag{6}$$

From these conditions, we find the constant A, B and C:

$$A = \frac{3u_{*s}d}{4D_{effective}}, \tag{7}$$

$$B = -\frac{u_{*s}d}{2D_{effective}}, \tag{8}$$

$$C = 0. \tag{9}$$

We can finally compute the stress at the bottom due to the presence of a non zero current:

$$\tau_{bTurb} = -\frac{1}{2}\rho u_{*s}^2 \tag{10}$$

it is independent on $D_{effective}$

Parameters
$C_D = 2.6 \cdot 10^{-3}$
$\rho_a = 1.2041$
$\rho = 998.2071$
$u_w = 1$
$D_{effective}$ is chosen so that $u_s = 0.1$ at the surface is imposed

$$D_{effective} = \frac{\tau_s d}{4\rho u_s}$$

Aspect ratio $L_z = 1$; $L_x = 40$

0.1 Resuspension by wind-induced current : case of the parabolic eddy viscosity distribution (Tsanis corrected)

Here we consider just a turbulent viscosity due to shear without thermal convection. For the moment we do not use this formulation.
To simulate the wind-driven flow in the shallow lake, we use a parabolic eddy viscosity distribution given in dimensional form by [3]:

$$D_t(z) = (\lambda u_{*s}d)\,(z + z_b/d)\,(1 - z/d + z_s/d) \tag{11}$$

where z_b and z_s are the bottom and surface characteristic lengths, respectively, d is the water depth, and λ is a constant to characterize the intensity of turbulence.
Since the shear stress should remain linear in z, one assumes a double-logarithmic velocity profile first proposed by [3]:

$$u(z) = Au_{*s}\ln(1 + z/z_b) + Bu_{*s}\ln(1 - z/(z_s + d)) + C. \tag{12}$$

z_b might depend on the plant rugosity and z_s the convection .
To determine the constants A, B and C, we set the following conditions [2]:

1. At the free surface, the shear stress is evaluated by the wind velocity:

$$\rho D(z) \frac{\partial u}{\partial z}\bigg|_{z=d} = \tau_s, \tag{13}$$

2. At the bottom, the velocity is equal to zero:

$$u(0) = 0. \tag{14}$$

3. The depth-averaged velocity equals zero:

$$\frac{1}{d}\int_0^d u(z)dz = 0 \tag{15}$$

To find the constants A and B we differentiate $u(z)$:

$$\frac{\partial u}{\partial z} = u_{*s}\left[\frac{A}{z_b + z} - \frac{B}{z_s + d - z}\right]. \tag{16}$$

So, given the condition 1, dividing both sides by ρu_{*s}, we obtain :

$$1 = \lambda z_{sh} A - \lambda(1 + z_{bh})B, \tag{17}$$

From the condition 3, one finds that

$$B = -\frac{q_1}{q_2}A. \tag{18}$$

Introducing the following abbreviations (note that they are different by the ones in [2]) :

$$z_{sh} = z_s/d, \tag{19}$$
$$z_{bh} = z_b/d, \tag{20}$$
$$p_1 = z_{sh}, \tag{21}$$
$$p_2 = (1 + z_{bh}), \tag{22}$$
$$q_1 = (1 + z_{bh})\ln(1 + 1/z_{bh}) - 1, \tag{23}$$
$$q_2 = z_{sh}\ln(1 + 1/z_{sh}) - 1, \tag{24}$$

we obtain

$$A_1 = \lambda A = \frac{q_2}{z_{sh}q_2 + (1 + z_{bh})q_1}, \tag{25}$$
$$B_1 = \lambda B = \frac{-q_1}{z_{sh}q_2 + (1 + z_{bh})q_1}, \tag{26}$$
$$C = 0. \tag{27}$$

Given these values for the coefficient A and B, we can finally compute the stress at the bottom due to the presence of a non zero current:

$$\tau_{bTurb} = \rho D_t(z) \frac{\partial u}{\partial z}\bigg|_{z=0} = \rho u_{*s}^2 \left[(1 + z_{sh}) A_1 - z_{bh} B_1 \right]. \tag{28}$$

Actually it is independent on λ.

Parameters
$C_D = 2.6 \cdot 10^{-3}$
$\rho_a = 1.2041$
$\rho = 998.2071$
$u_w = 1$

and

$\lambda = 0.35$
$z_s/d = 2.2 \cdot 10^{-4}$
$z_b/d = 2 \cdot 10^{-3}$
$u_{*s} =$

Aspect ratio $L_z = 1$; $L_x = 40$

References

[1] Yang, Y., Straatman, A. G., Hangan, H., Yanful, E. K., An engineering model for countercurrent flow under wind-induced waves and current. Environ. Fluid. Mech. 8, 2008, pp. 19-29.

[2] Wu, J., Tsanis, I.K., Numerical study of wind-induced water currents. J. Hydr. Engrg., ASCE, 121 (5), 1995, pp. 388-395.

[3] Tsanis, I.K., Numerical study of wind-induced water currents. J. Hydr. Engrg.,ASCE, 115 (8), 1989, pp. 1113-1134.