Quadtree-adaptive tsunami modelling

Stéphane Popinet

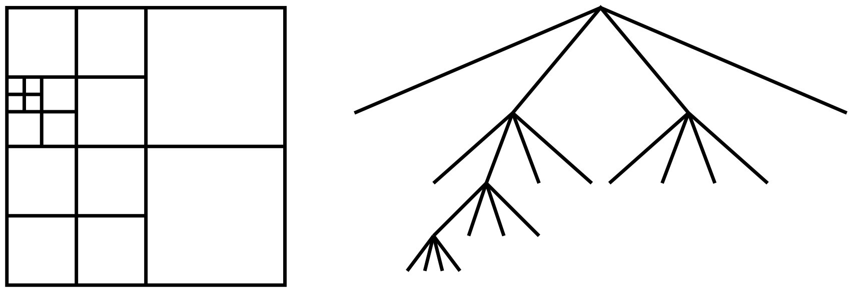
National Institute of Water and Atmospheric research

Wellington, New Zealand

Institut d'Alembert, Université Pierre et Marie Curie

Paris, France

Adaptive solutions of Partial Differential Equations



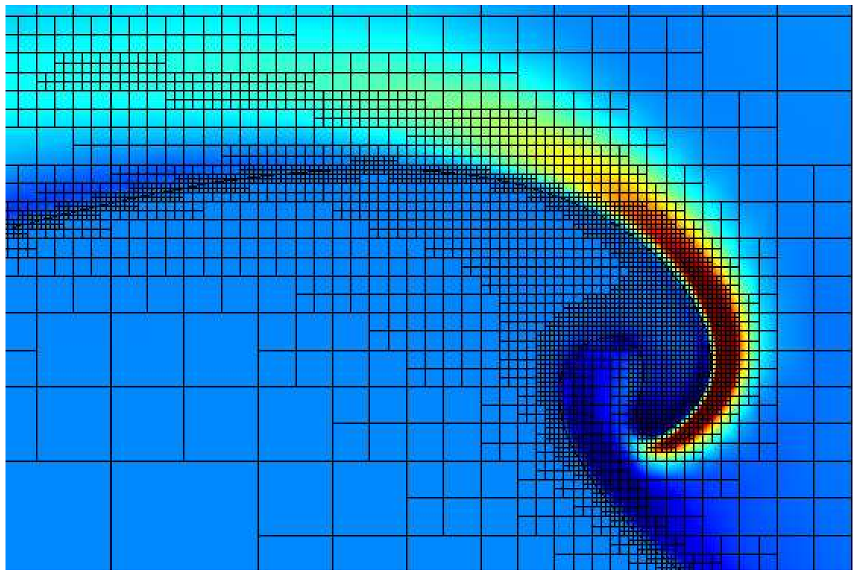
- Gerris Flow Solver

  `gfs.sf.net`

- Navier–Stokes, Euler, Saint-Venant

  etc...

- Adaptive quad/octree discretisation
- Free Software (GPL)
- Parallel with dynamic load-balancing
- Popinet (2003, 2009), *JCP*



“The curse of dimensionality”

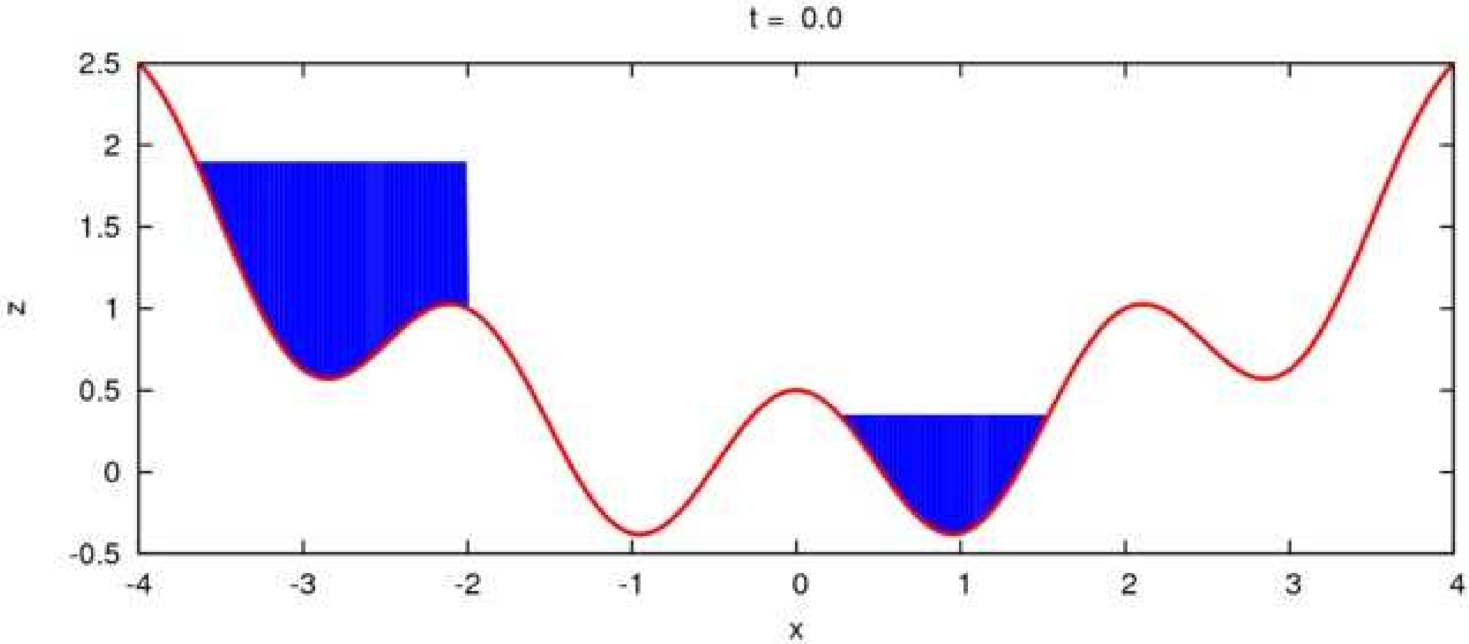
or is adaptive mesh refinement necessary?

- The universe has (at least) four dimensions
- Using regular Cartesian grids, solution costs scale like

$$C\Delta^{-4}$$

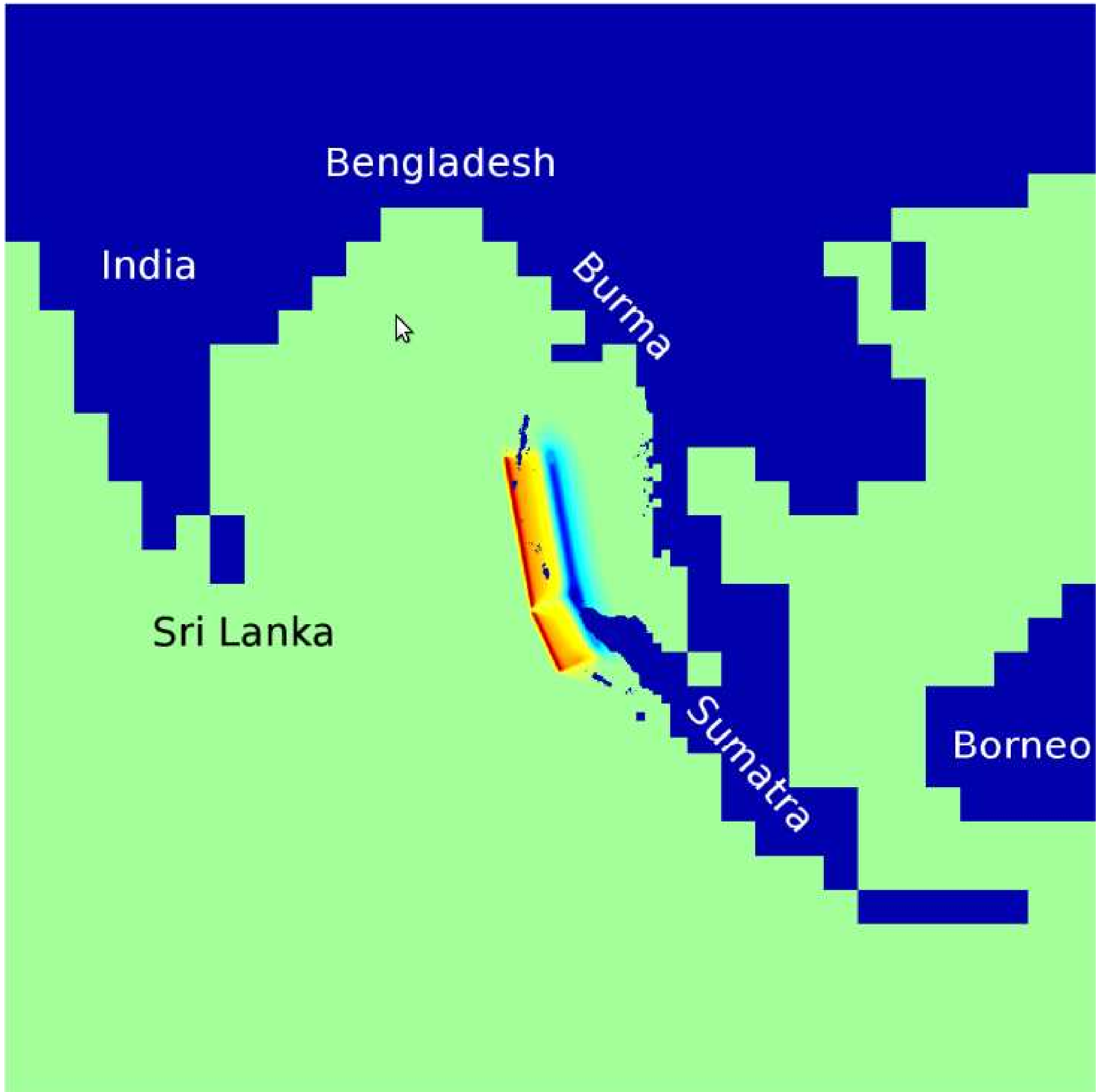
  with C a constant and Δ the spatial resolution in each dimension
- Just buy bigger computers! A 100-fold increase in computing power will buy you a $\sqrt[4]{100} \approx 3$-fold increase in resolution... (assuming C does not increase)
- Can adaptive methods break the spell?

The Saint-Venant equations



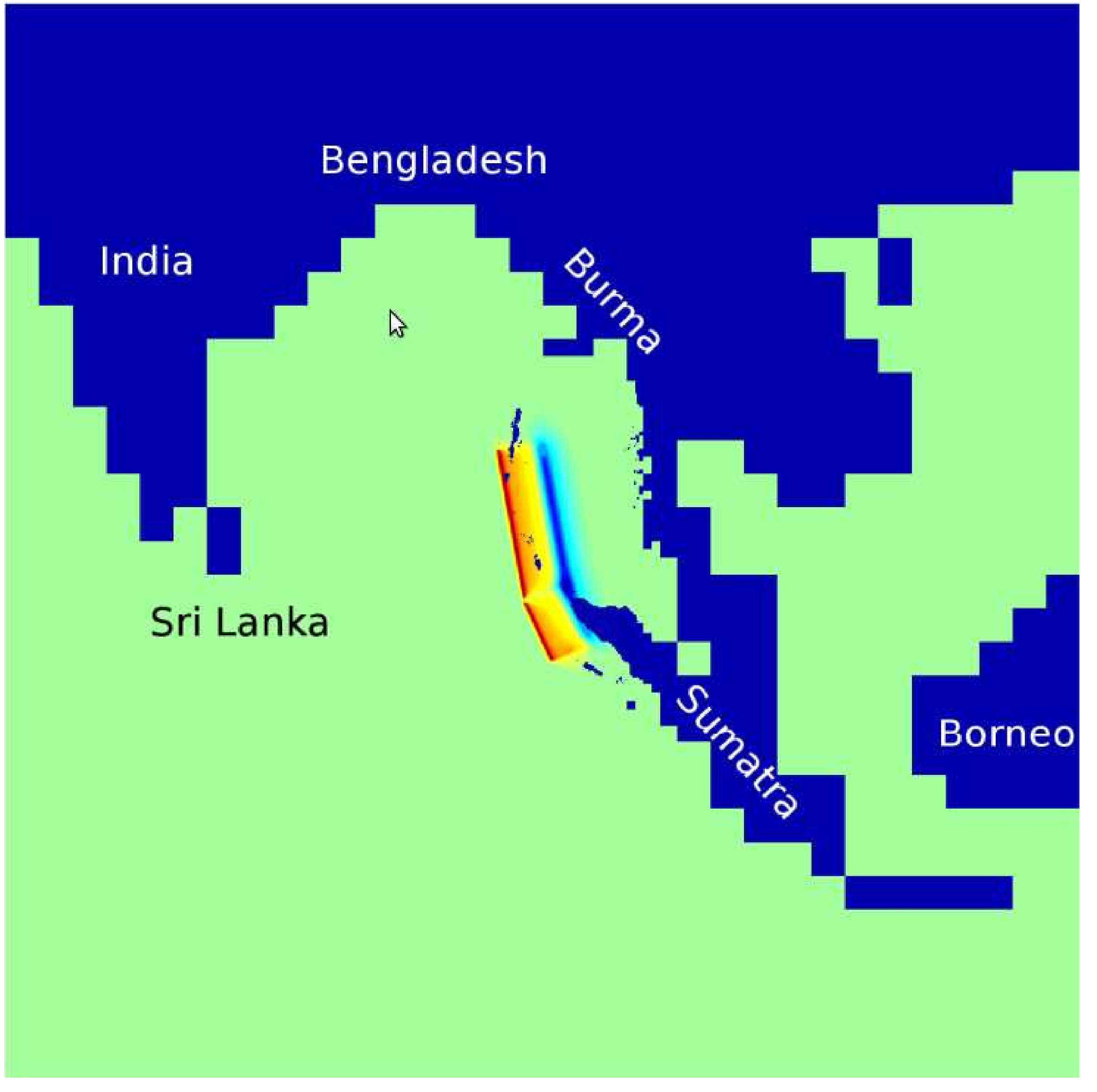
- Godunov-type finite-volume scheme
- HLLC approximate Riemann solver
- Wetting/drying, hydrostatic equilibrium: scheme of Audusse et al (2004)

2004 Indian ocean tsunami



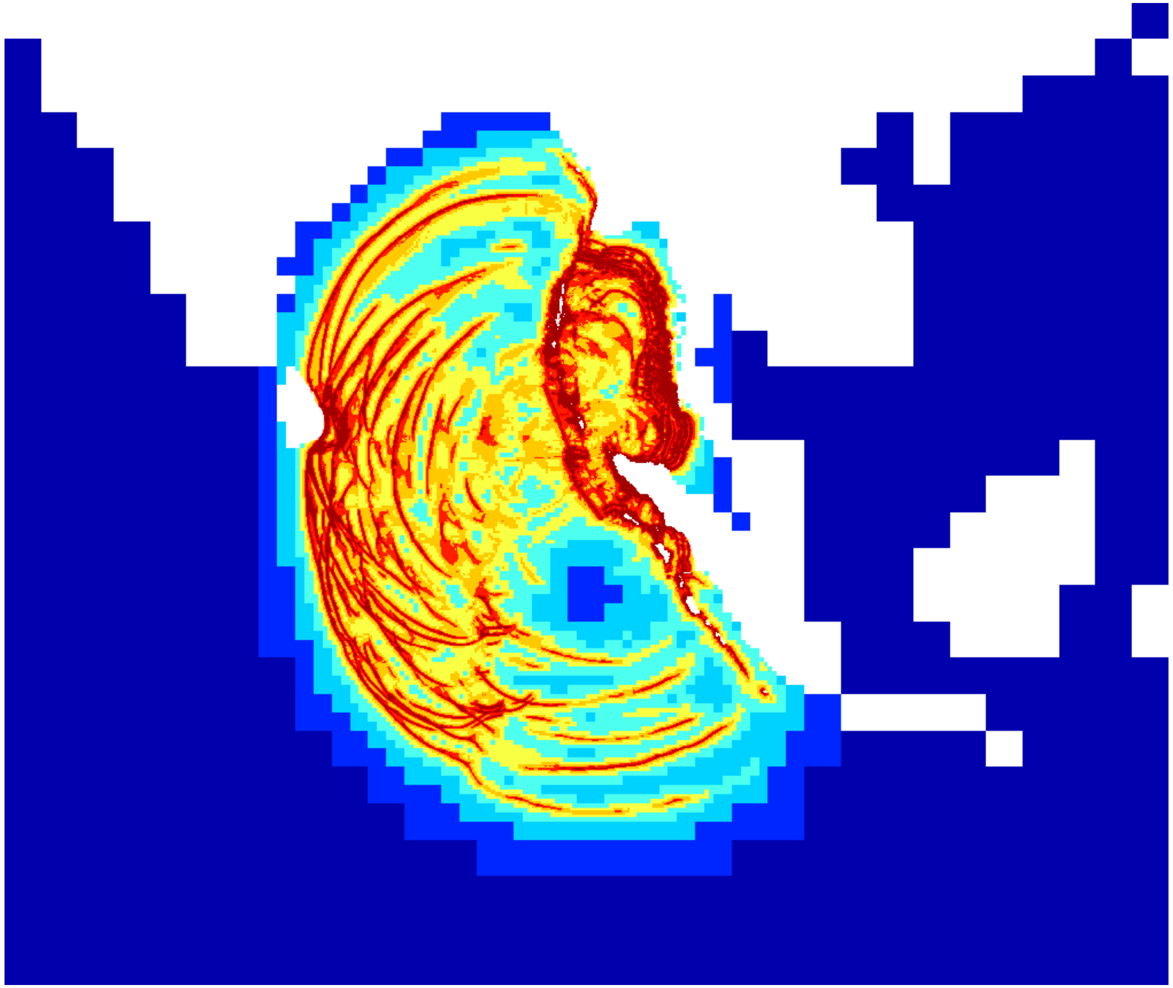
Staggered fault displacement model (5 segments)

2004 Indian ocean tsunami



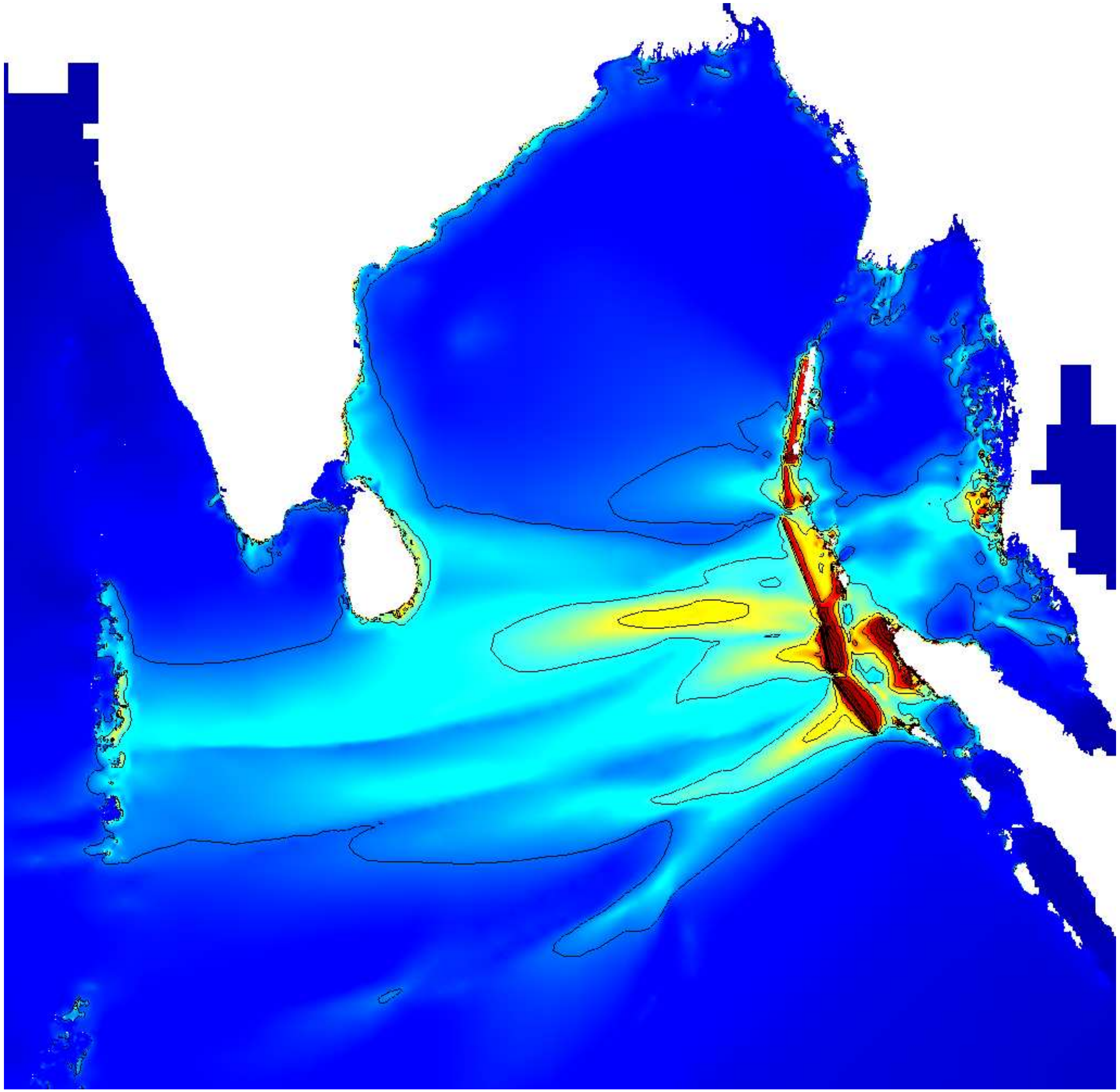
$1 \text{ km} \leq \text{Spatial resolution} \leq 150 \text{ km}$

Adaptivity

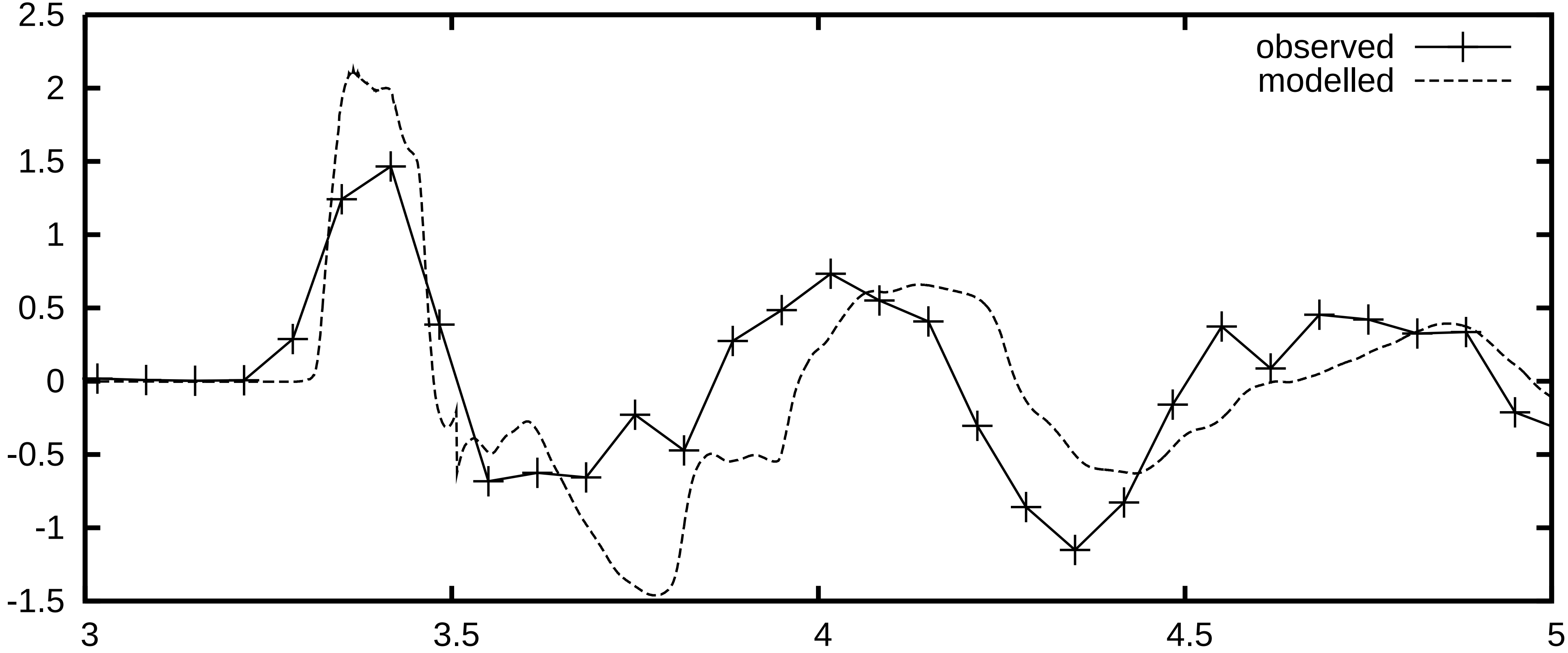


Truncation error of the wave height < 5 cm

Maximum wave height



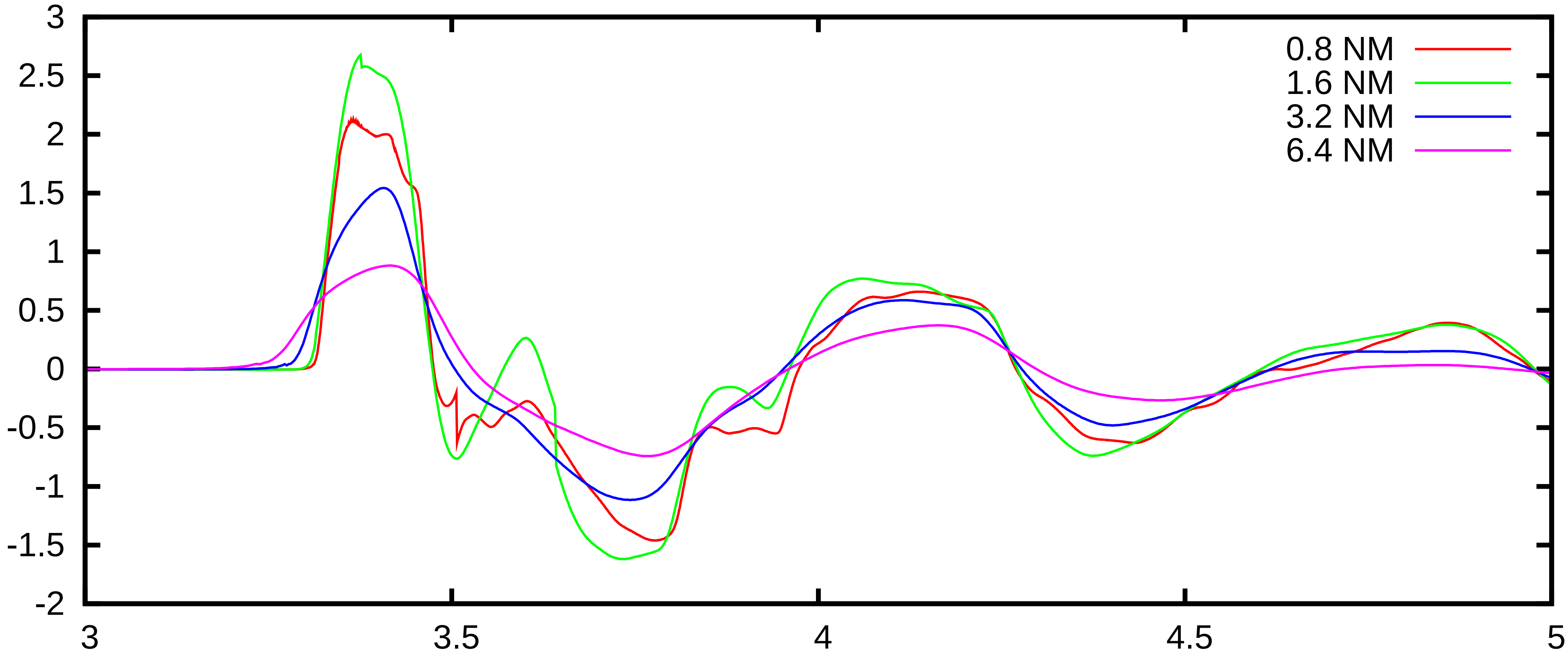
Comparison with field data



Tide gauge at Male, Maldives

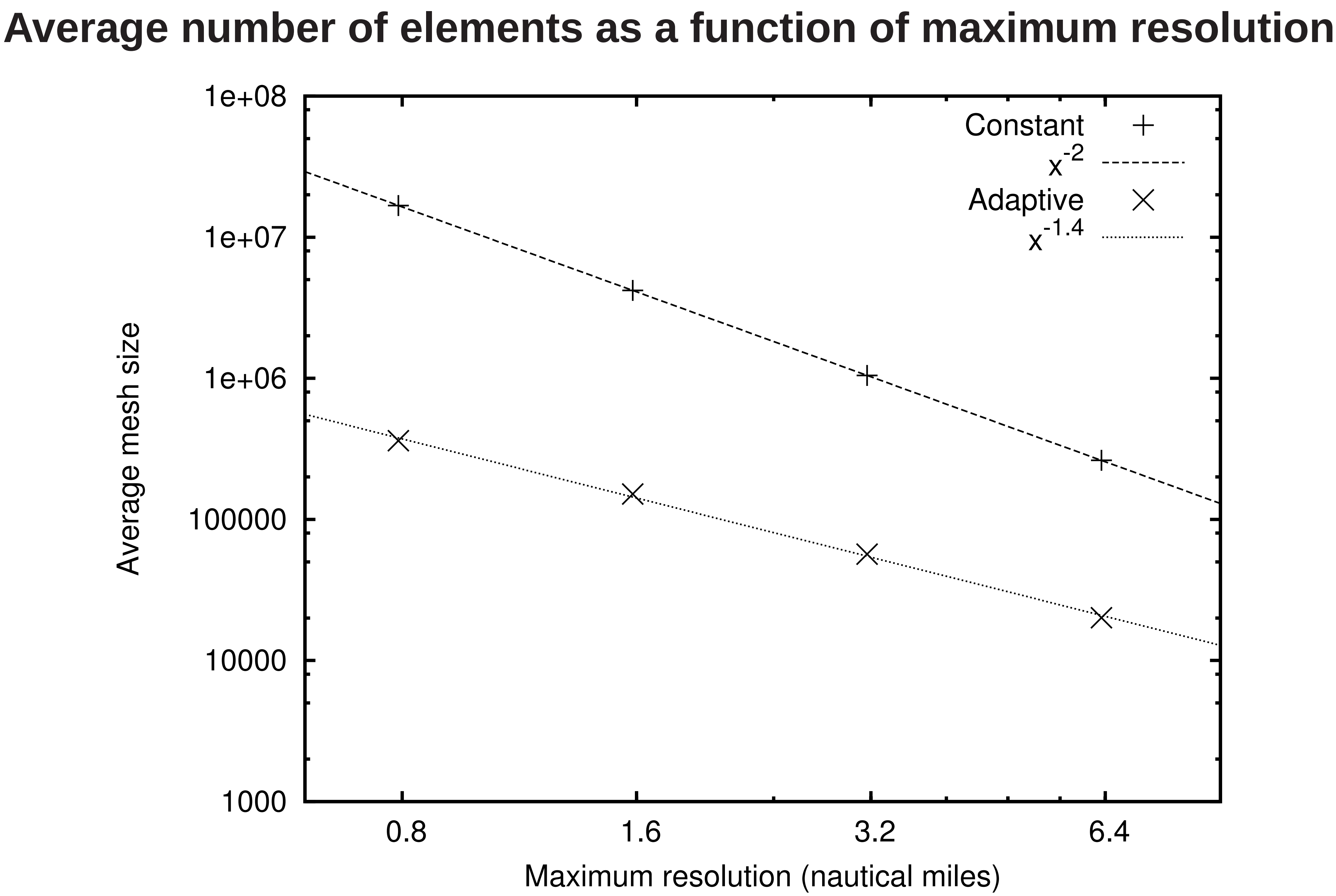
Time in hours, wave height in metres

Effect of spatial resolution



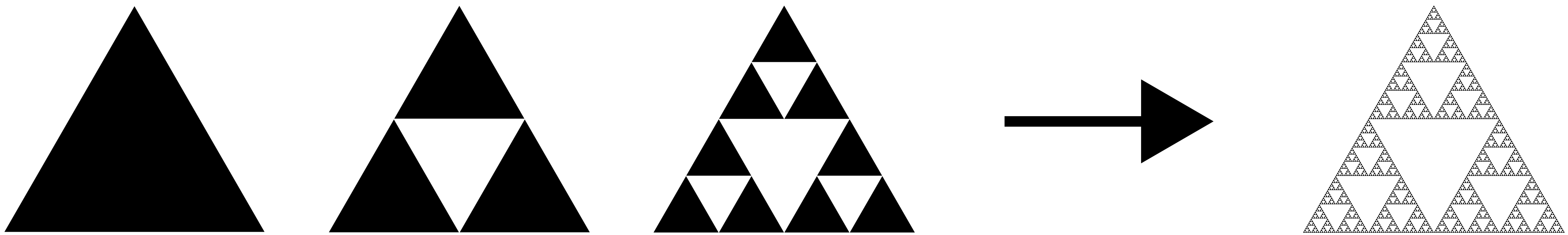
Tide gauge at Male, Maldives

Time in hours, wave height in metres


Average number of elements as a function of maximum resolution
Constant
x^-2
Adaptive
x^-1.4
Average mesh size
1e+08
1e+07
1e+06
100000
10000
1000
0.8
1.6
3.2
6.4
Maximum resolution (nautical miles)

Connection with fractal dimension

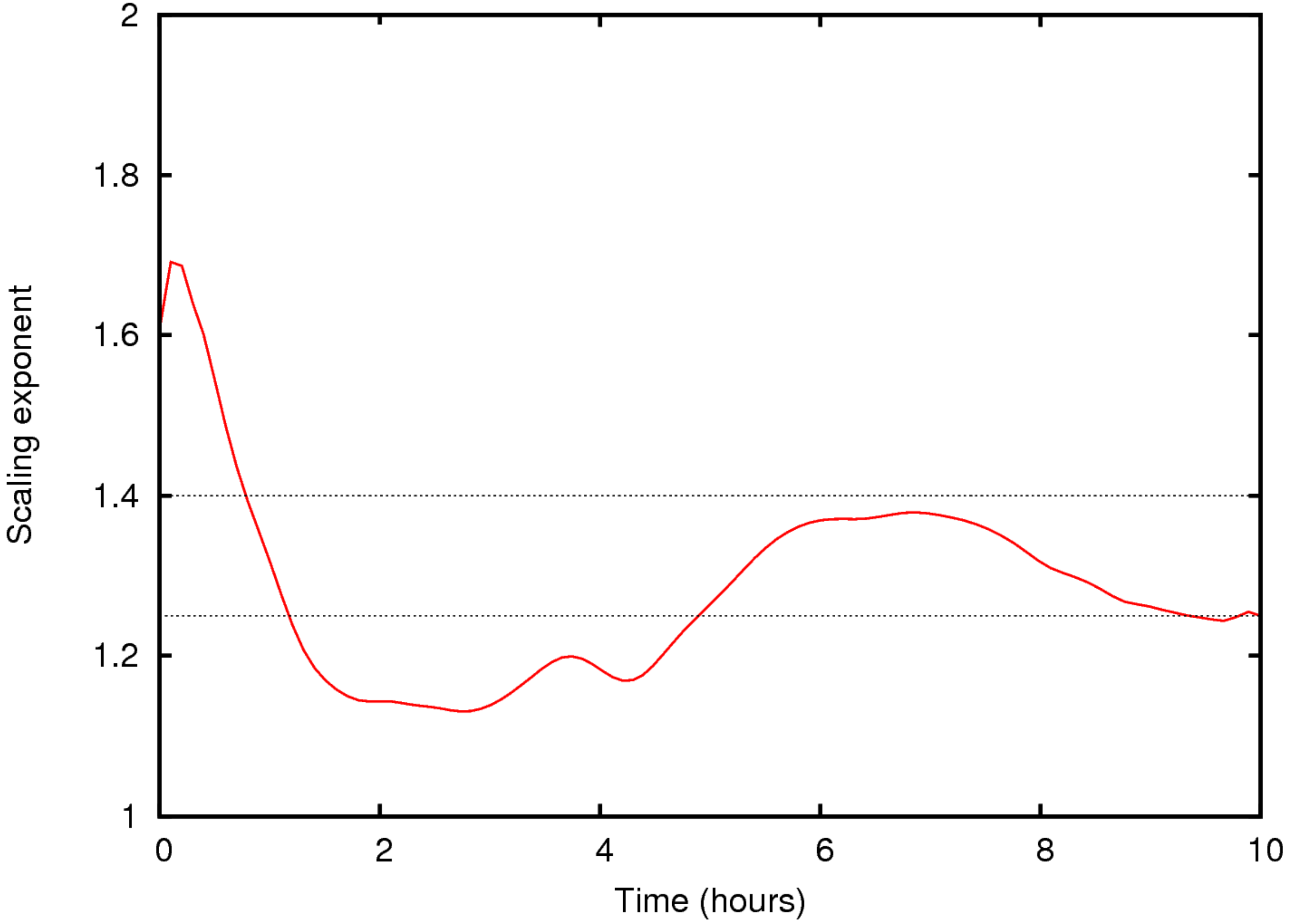
Classical example: the Sierpinski triangle



has a fractal (Minkowski–Bouligand or “box-counting” or “information”) dimension of ≈ 1.6.

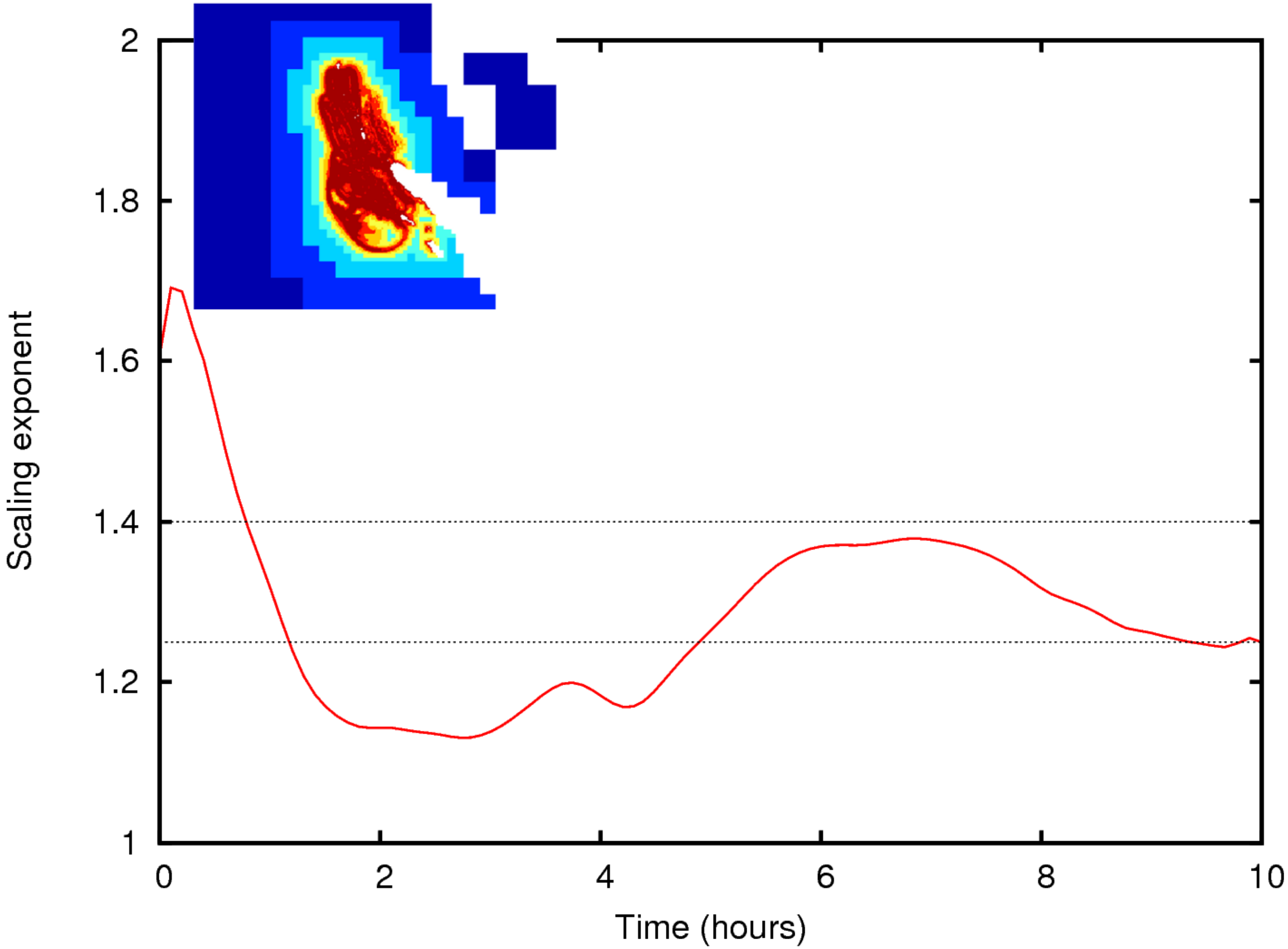
In other words, the cost of describing such an object using quadtrees would scale as $\Delta^{-1.6}$ not Δ^{-2}.

Evolution of the scaling exponent with time



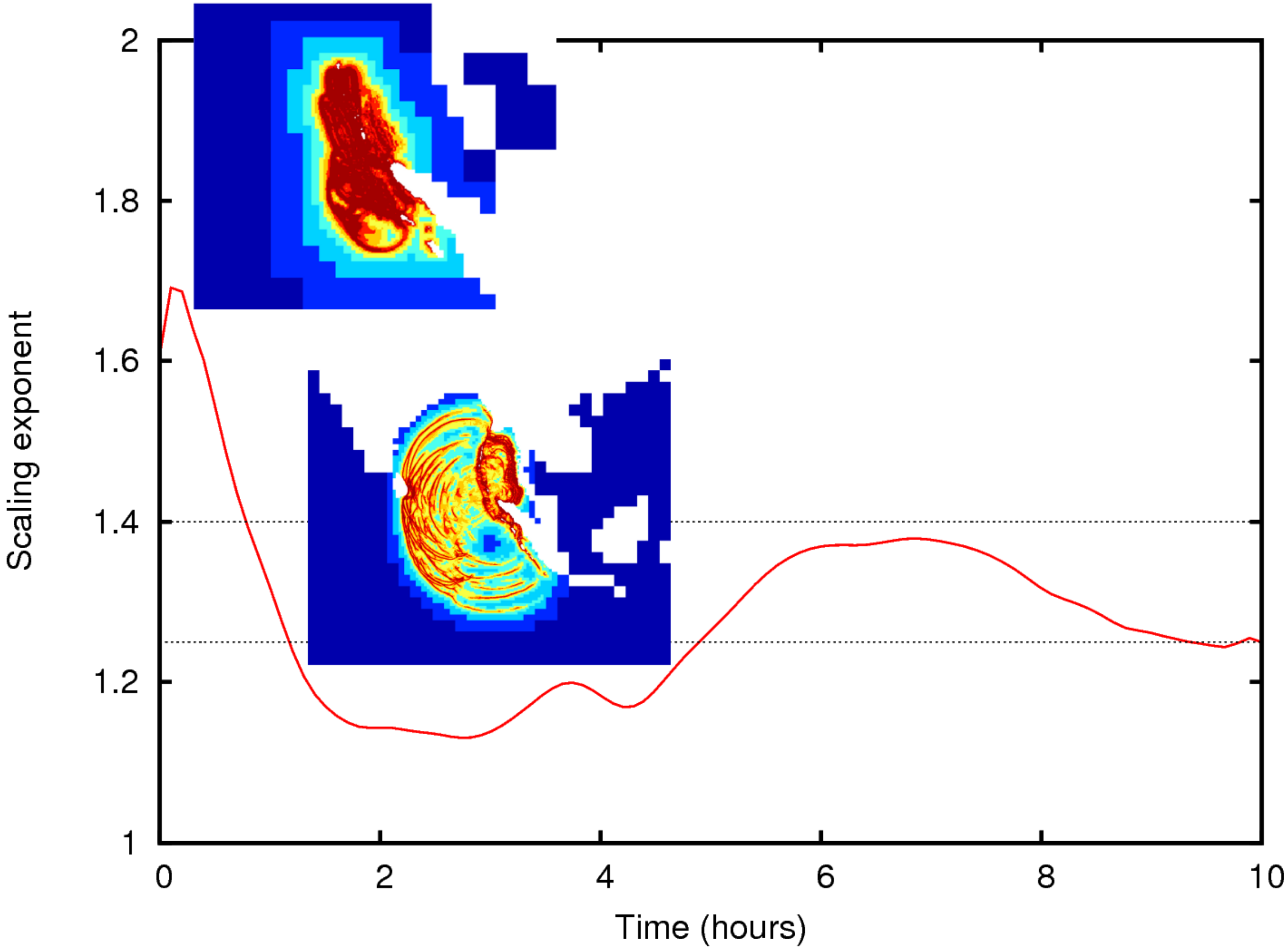
Mandelbrot, *How long is the coast of Britain?*, Science, 1967

Evolution of the scaling exponent with time



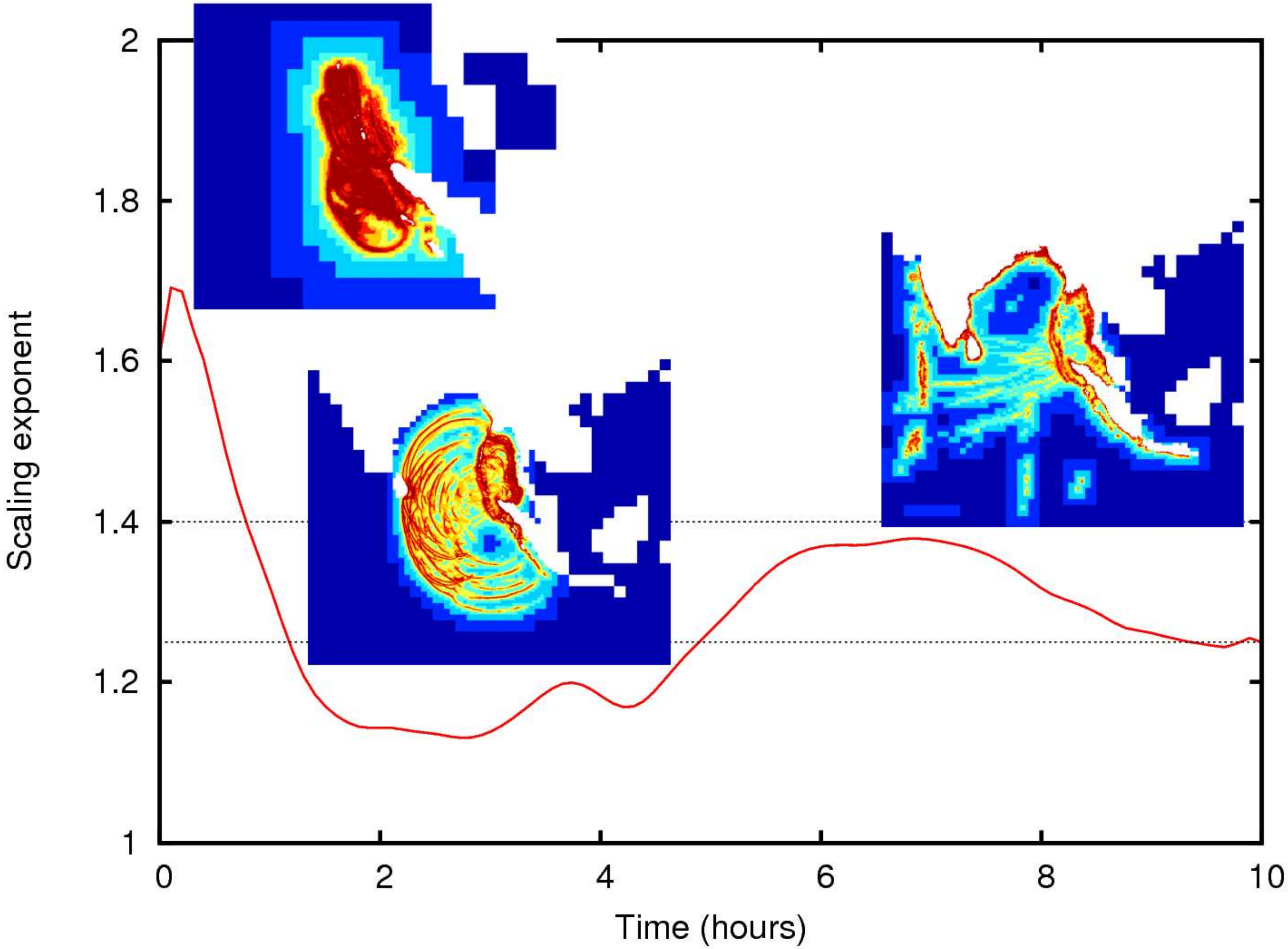
Mandelbrot, *How long is the coast of Britain?*, Science, 1967

Evolution of the scaling exponent with time



Mandelbrot, *How long is the coast of Britain?*, Science, 1967

Evolution of the scaling exponent with time



Mandelbrot, *How long is the coast of Britain?*, Science, 1967

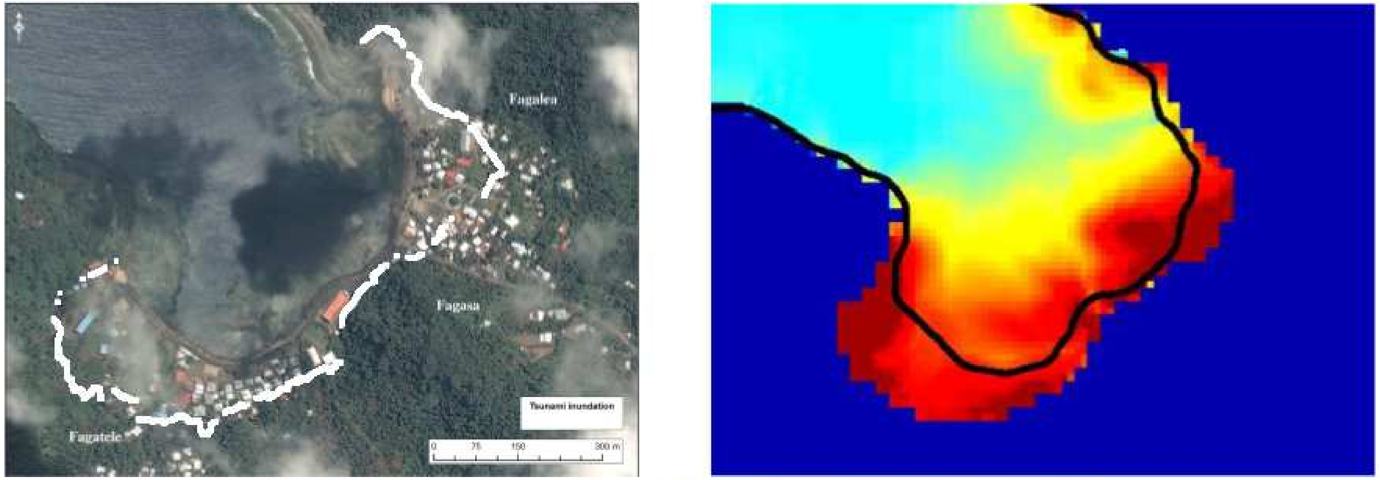
Conclusions

- Accurate and fast solutions for multiscale Saint-Venant problems
- Adaptivity changes the scaling of computing costs: $C\Delta^{-d}$, d is now smaller than the number of dimensions
- This conclusion extends to a range of problems (not just Saint-Venant)
- See also poster for the Tōhoku tsunami

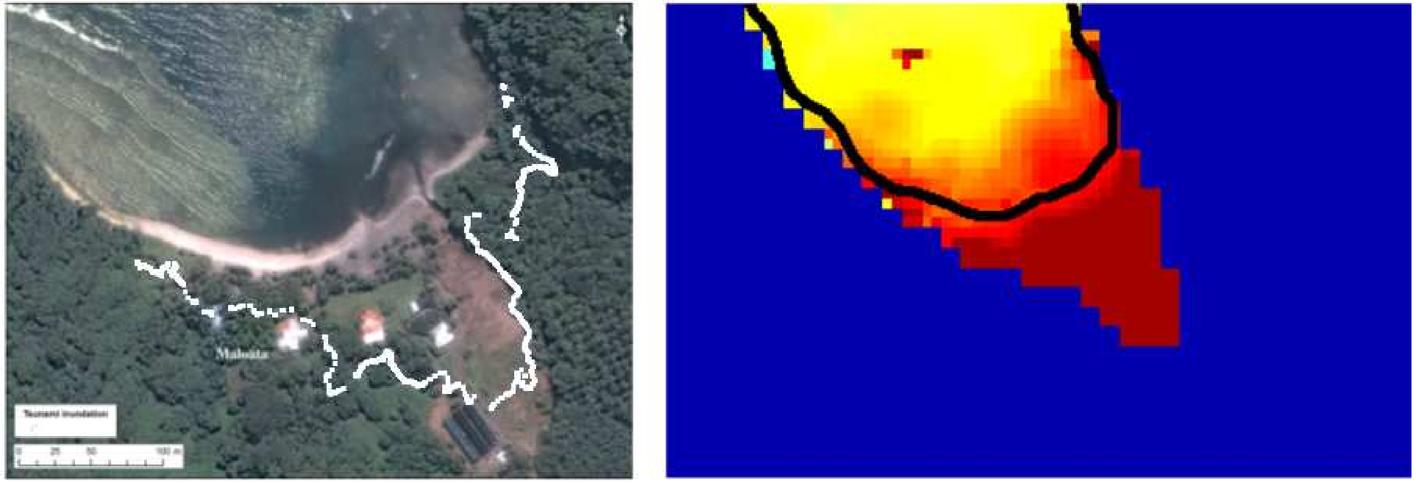
Work in progress

- There is a close link between the physical scale-distribution of (fluid dynamics) problems and the scaling of computing costs: this needs to be explored to make the most of adaptive methods

Inundation at Tutuila, American Samoa, 2009



Fagasa



Maloata

10 m $\leq$ Spatial resolution $\leq$ 82 km

Simulated domain $\approx$ (3000 km)^2

Maximum runups on shoreline

Locations	Model	Field surveys
Aceh (N coast), Indonesia	8.25	10–16
Aceh (W coast), Indonesia	17.60	24–35
Galle, Sri Lanka	3.16	2–3
SE coast, Sri Lanka	5.60	5–10
Chennai, India	3.01	2–3
Nagappaattinam, India	3.20	2–3.5
Kamala Bch., Phuket, Thailand	5.95	4.5–5.3