# What is the liquid jet atomization problem?
# Will Gerris help us solve it?

*Stéphane Zaleski*

*Institut Jean Le Rond d'Alembert,*
*Université Pierre et Marie Curie UPMC – Paris 6*

http://www.lmm.jussieu.fr/~zaleski

UPMC PARIS UNIVERSITAS

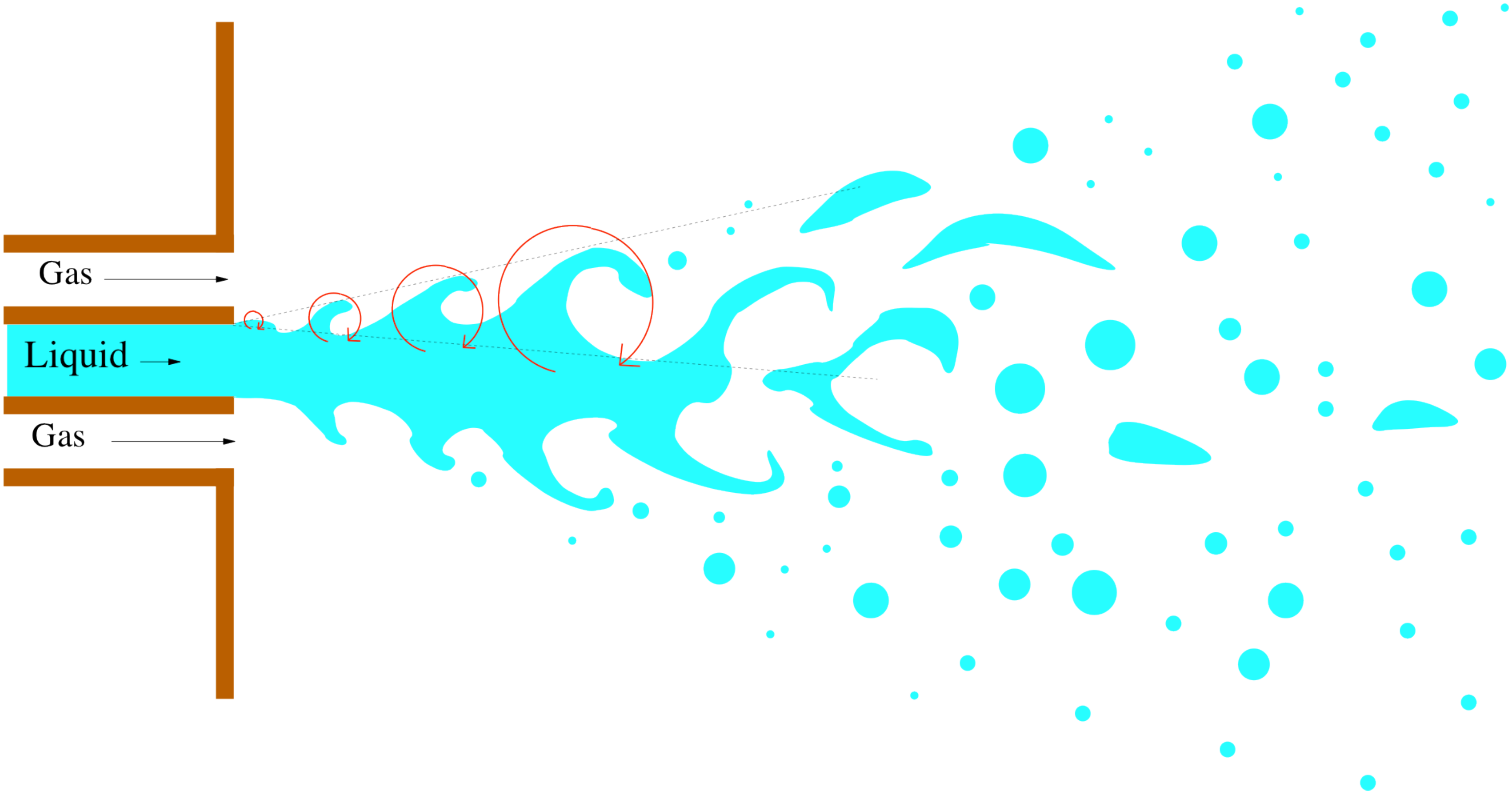
cnrs

## Collaborators past and present on atomization.

Jie Li (BPI, Cambridge), Phil Yecko (Montclair, NJ), Thomas Boeck (Ilmenau), Jose-Maria Fullana (d'Alembert), Ruben Scardovelli (Bologne), Stéphane Popinet (d'Alembert), Pascal Ray (d'Alembert), Luis Lemoyne (Nevers), Gaurav Tomar (Bangalore), Daniel Fuster (d'Alembert), Anne Bagué (ONERA), Jérôme Hoepffner (d'Alembert), Ralph Blumenthal (TUM), Annagrazia Orrazo (Naples).

## Current Student

Gilou Agbaglah

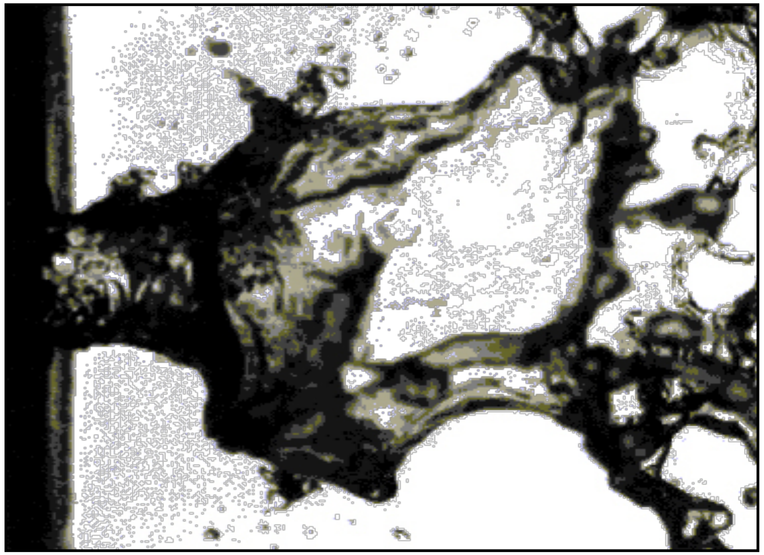



coflowing jet atomization (rocket engines, Formula 1 racing cars)

Interpretation 1: the simulation of liquid jet atomisation requires enormous ressources. It is thus a grand challenge similar to the DNS of turbulent flow. The one with the most robust code and the biggest computer wins.

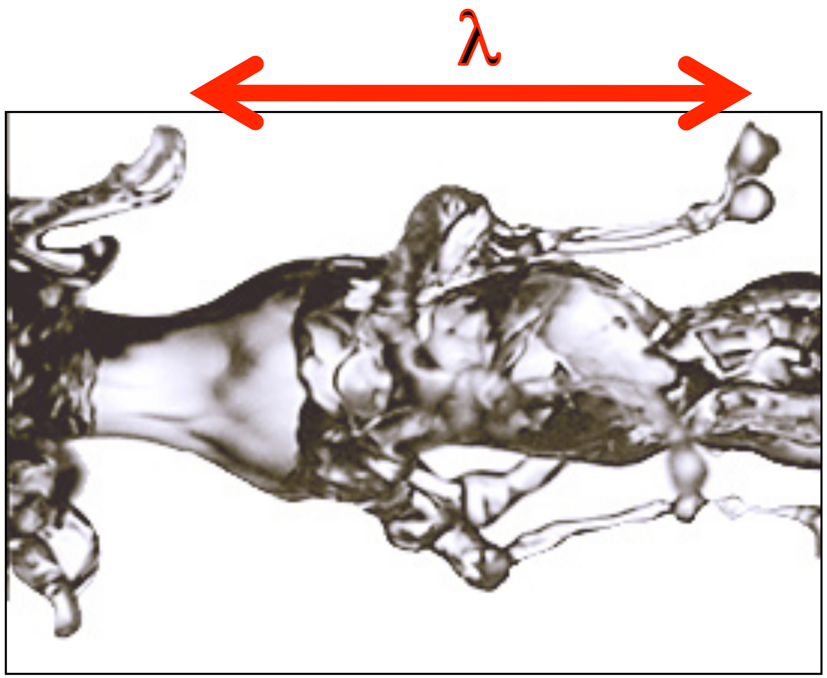
Interpretation 2: A series of mechanisms are involved in the breakup of the jet and generation of the droplets. Explaining these mechanisms and making quantitative predictions solves the problem.

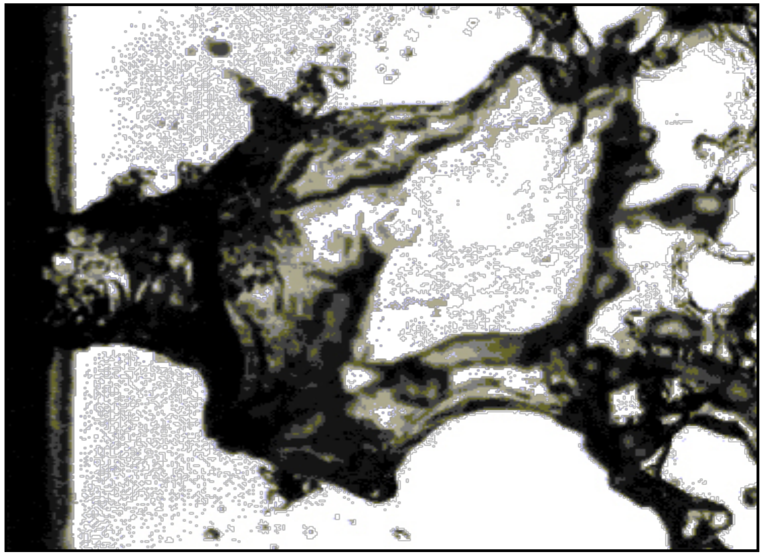
Atomization of co-axial jets : experiments with air/water jets



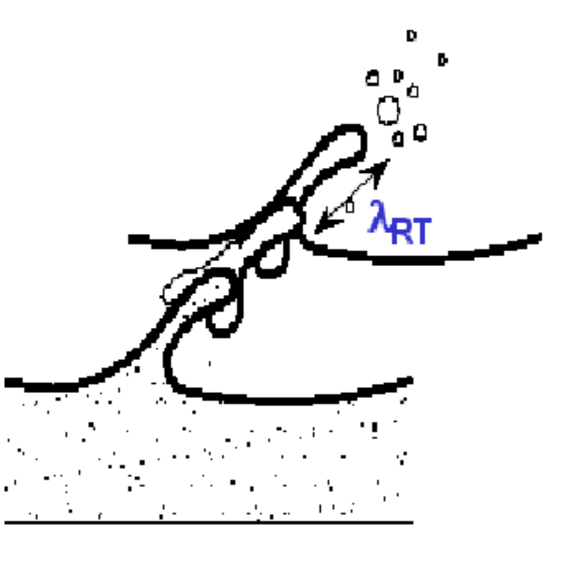
Lasheras, Hopfinger, Villermaux, Raynal, Cartellier …,
(San Diego, Grenoble and Marseille)

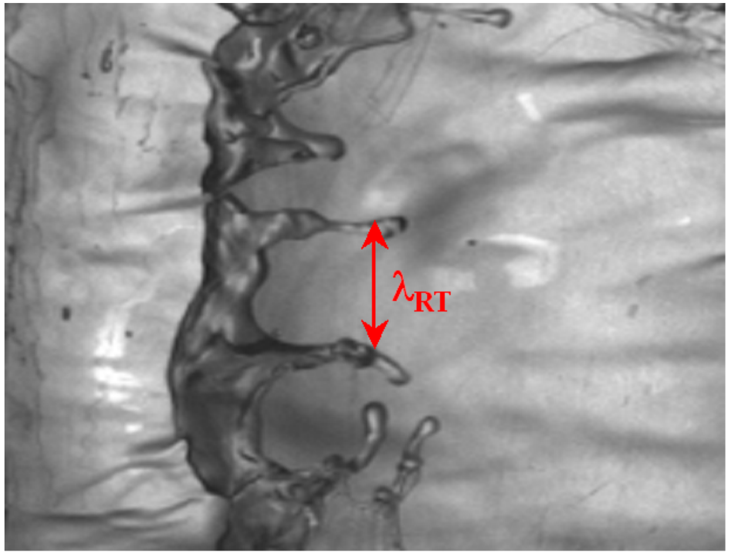
Problem 1: predict the transverse wavelength λ or the frequency $f$



## an atomizing sheet seen from above:



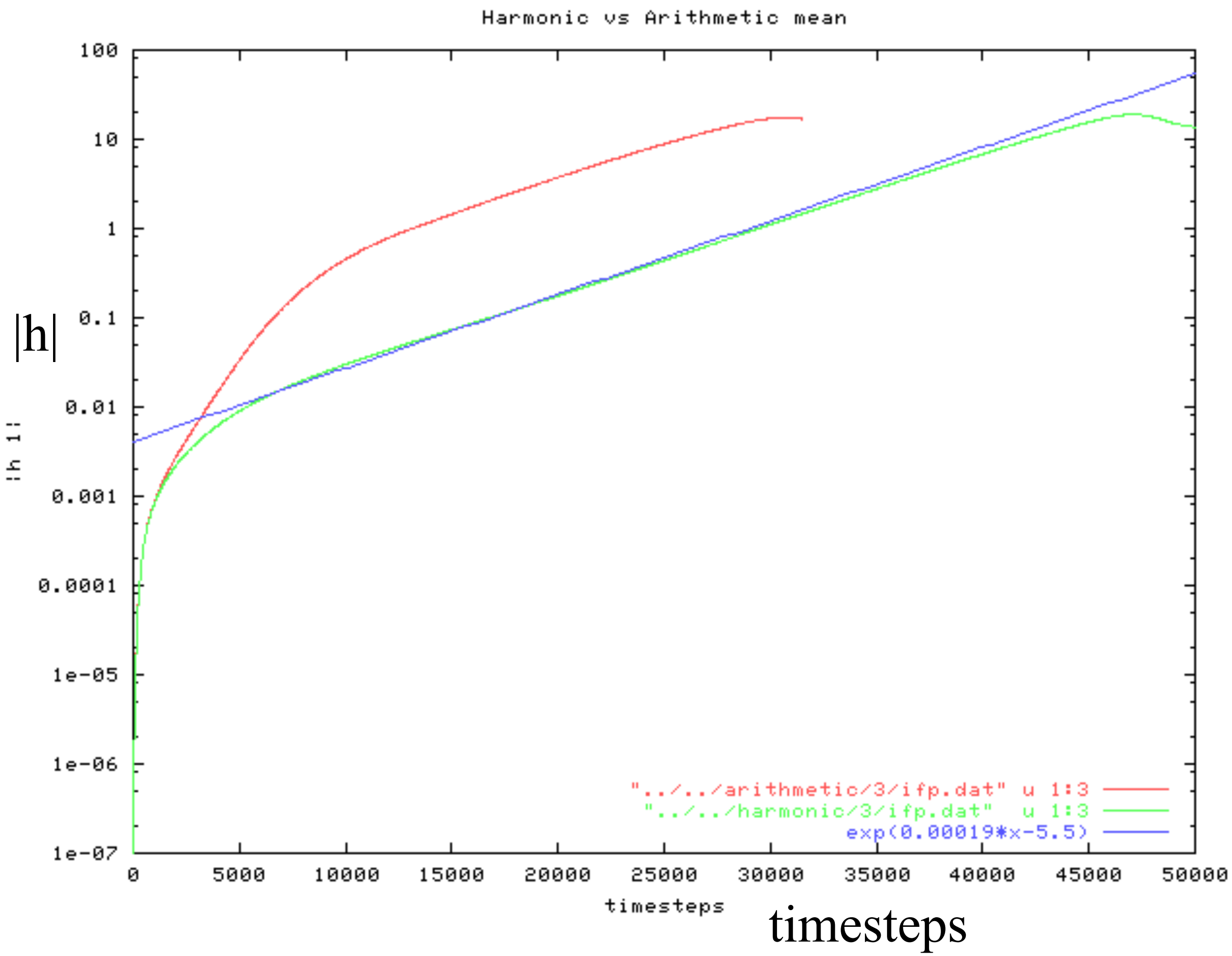

Photograph: Cartellier and Matas

Problem 2 : predict $\lambda_{RT}$ . $\lambda_{RT}$ refers to the theory of Cartellier and Hopfinger but in fact several "Rayleigh-Taylor" mechanisms have been suggested in the litterature.

Kelvin-Helmholtz instability

perform simulation of unstable shear flow

# Full DNS and Orr-Sommerfeld agree




Blue : theory
Green: computation with harmonic mean
Red : computation with arithmetic mean

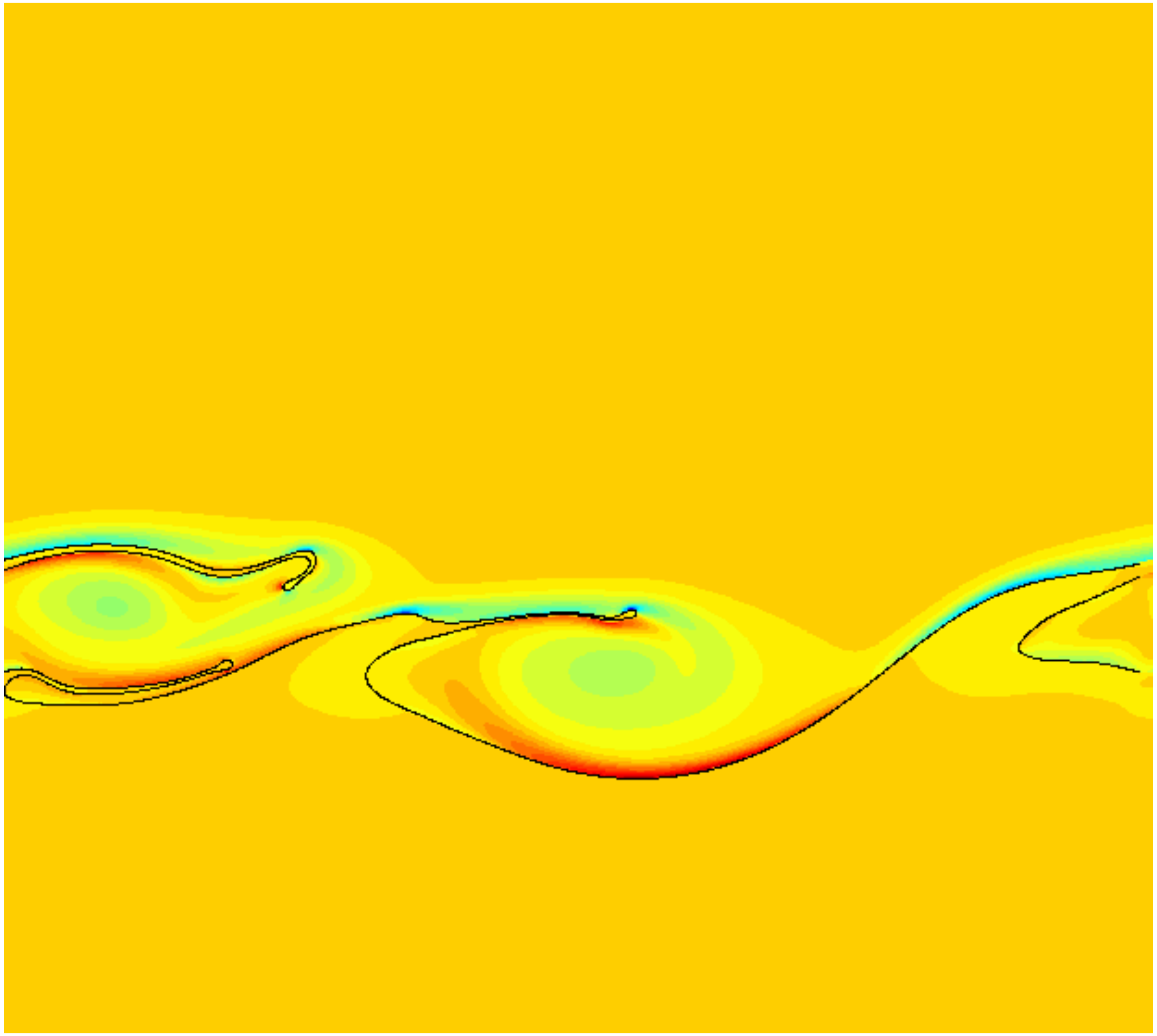
Boeck & Zaleski PoF 2005, Boeck, Li, Lopez-Pages, Yecko & Zaleski TCFD 2007, Bagué, Fuster, Popinet, Scardovelli and Zaleski, PoF 2010.

| Case | $m$ | $r$ | $\delta_l/L_l$ | $\delta_g/L_g$ | $Re_l$ | $Re_g$ | $We_l$ | $We_g$ |
|---|---|---|---|---|---|---|---|---|
| $A$ | 0.1 | 1 | 1/6 | 1/6 | 200 | 2000 | $\infty$ | $\infty$ |
| $B$ | 0.1 | 1 | 1/6 | 1/6 | 200 | 2000 | 10 | 10 |
| $C$ | 0.99 | 0.1 | 1/6 | 1/6 | 19800 | 2000 | $\infty$ | $\infty$ |
| $D$ | 0.99 | 0.1 | 1/6 | 1/6 | 19800 | 2000 | 100 | 10 |

Error level in percentage points :

| code | GERRIS | | | | | SURFER | | | |
|---|---|---|---|---|---|---|---|---|---|
| case | 16 | 32 | 64 | 128 | 256 | 32 | 64 | 128 | 256 |
| $A$ | - | 21.33 | 10.74 | 3.50 | 1.5 | - | 22.87 | 10.47 | 5.82 |
| $B$ | - | 7.30 | 1.28 | 0.48 | 1.04 | - | 29.43 | 20.47 | 13.94 |
| $C$ | 3.00 | 1.17 | 0.24 | 0.14 | 0.09 | 33.21 | 16.72 | 8.63 | 4.32 |
| $D$ | 3.98 | 1.39 | 0.76 | 0.07 | 0.54 | 33.49 | 16.1 | 8.67 | 6.19 |

Gerris is much more accurate than Surfer but the error stops decreasing for 256 !



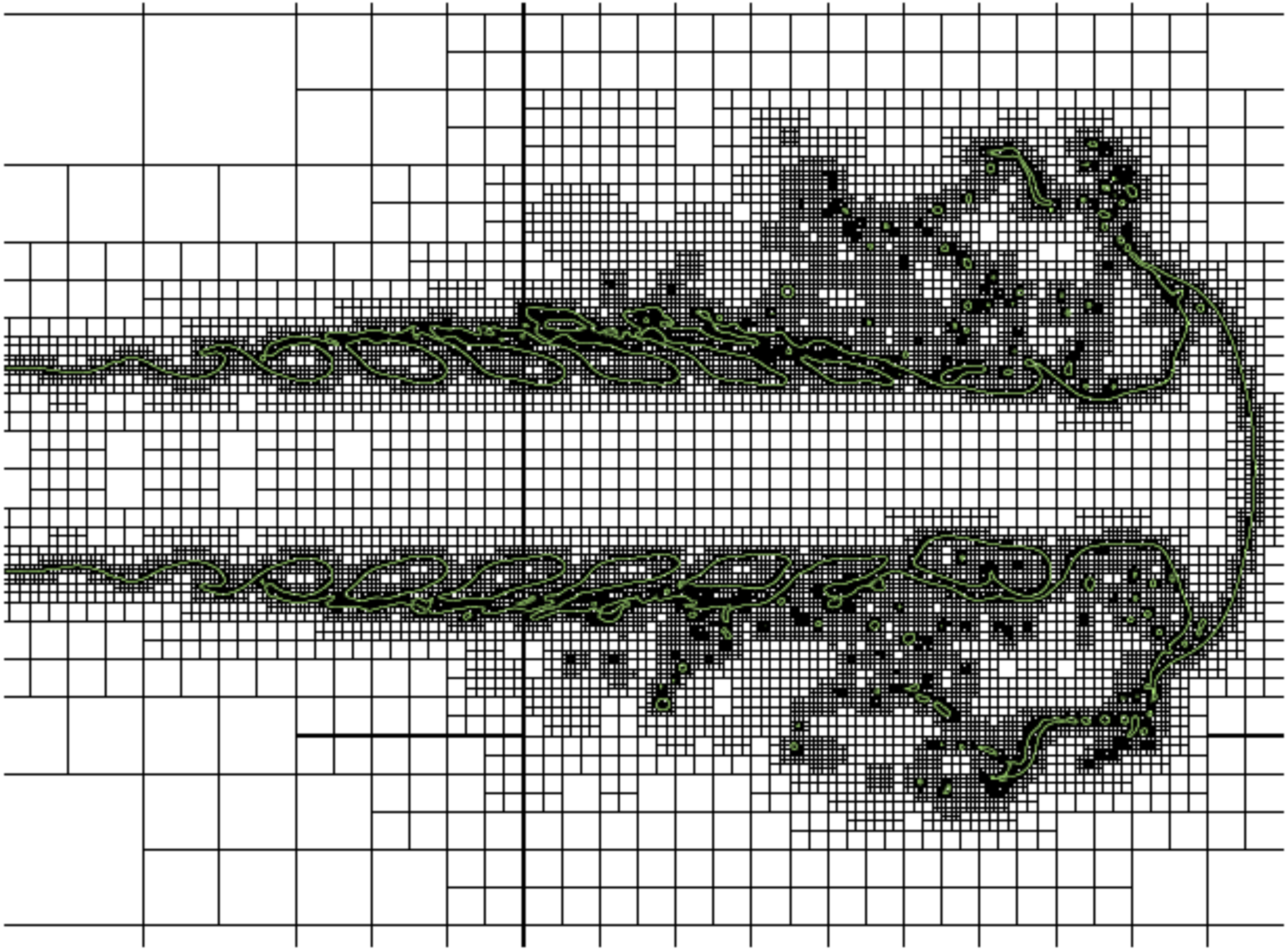
Re_g = 4000,
We_g = 4000
r = 10
m = 0.025
vorticity.

Diesel
jet
conditions

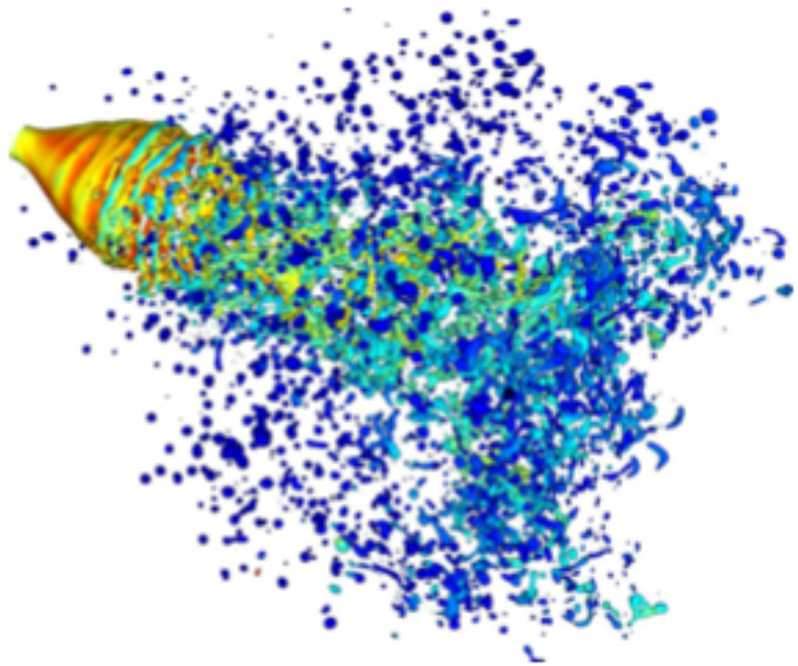
Bagué
Popinet
Yarlagadda

$\rho_l/\rho_g = 28$
$We_l = 11600$
$Re_l = 5800$

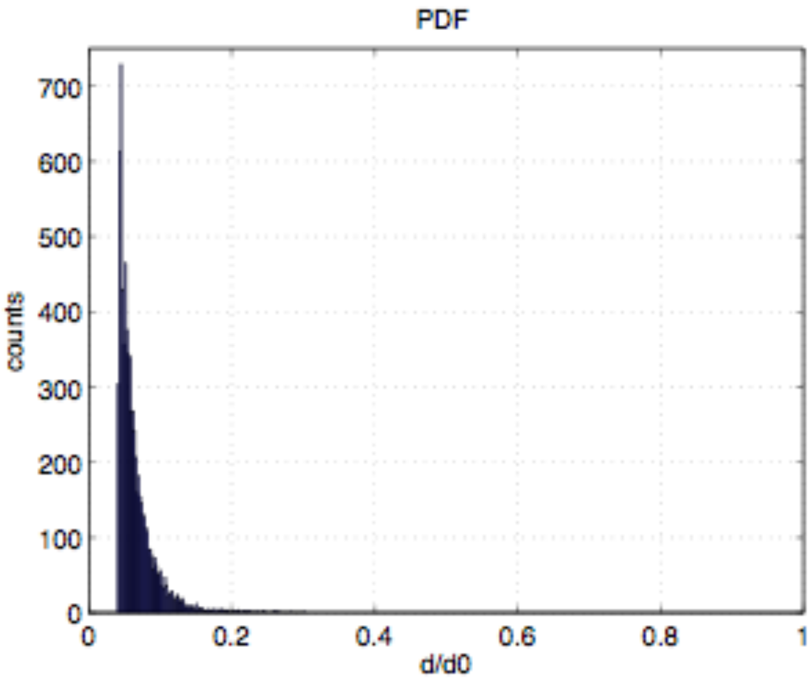
It is a
Gerris
example
now !

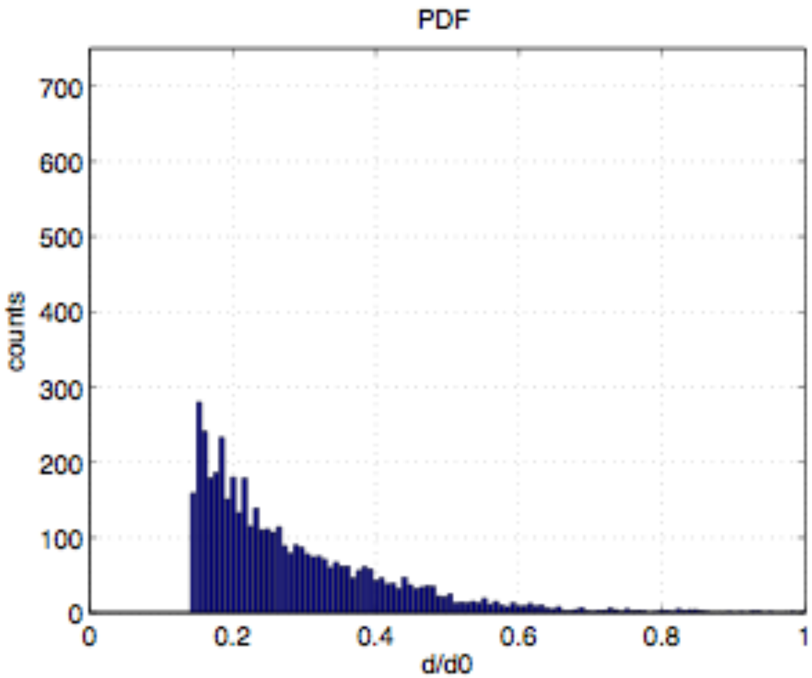


More realistic engineering: 3D conical jet



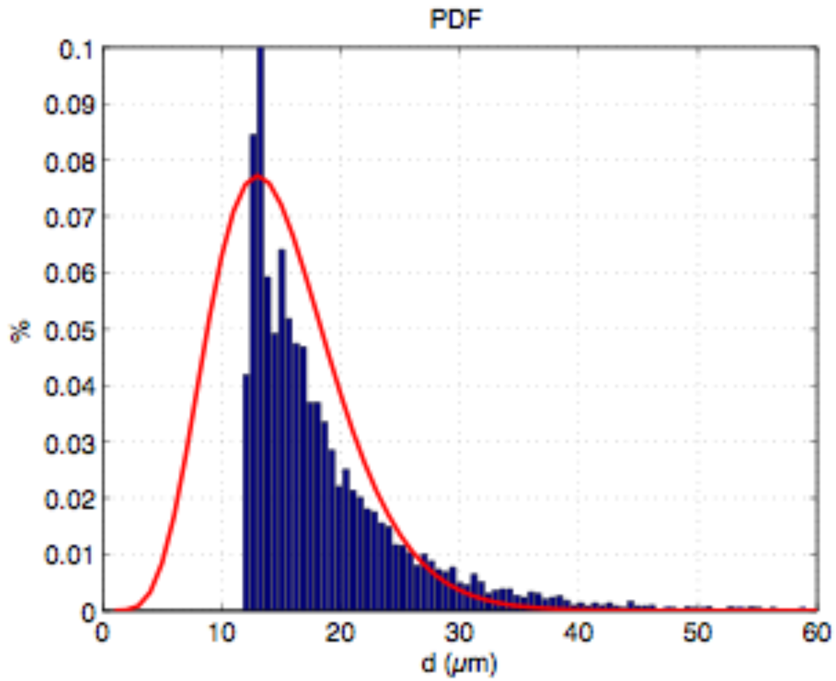
Distribution of droplet sizes (PDF) depends on grid size ! ☹



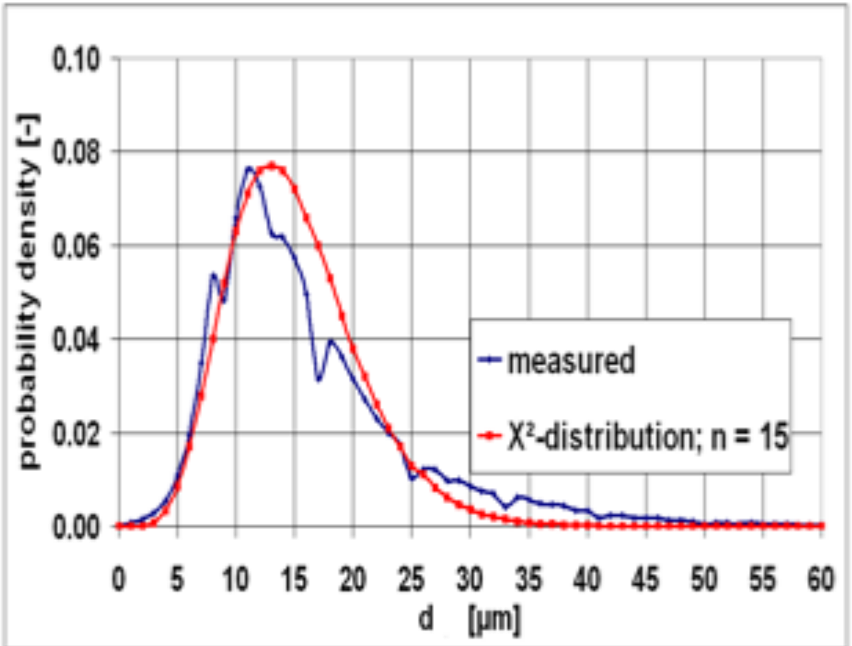

$\Delta x = 9$ microns

$\Delta x = 28$ microns




But agrees with experiment at finest resolution ☺

dx = 9 microns




experiment

Clearly, we are not world champions in terms of hardware usage

.

Shinjo and Uemura JAXA supercomputer

$\Delta x$ = 350 nm

5760 cores

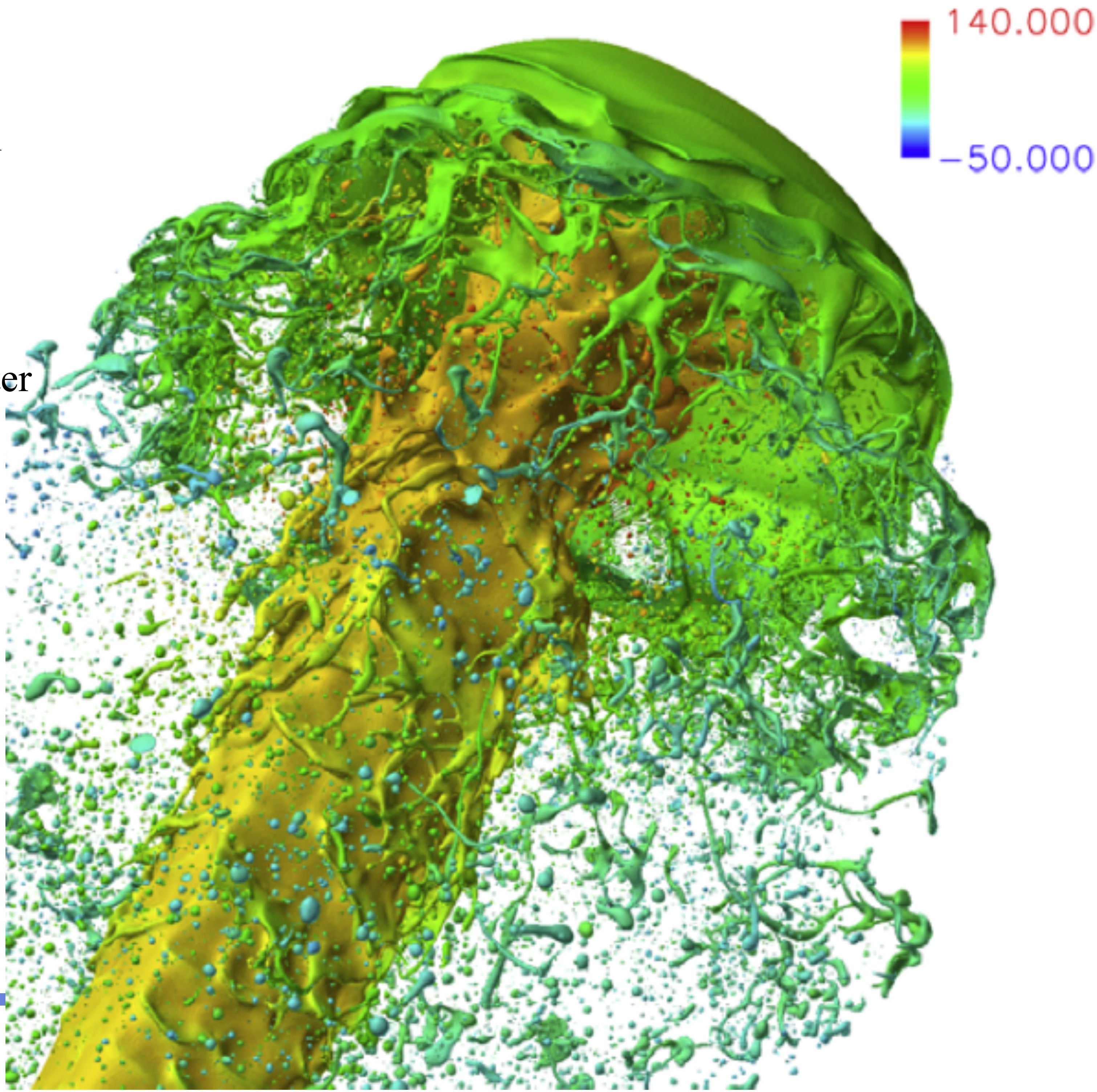
for 410 hours

6 billion grid points (uniform)

We=14000

Re=1470

U = 100 m/s

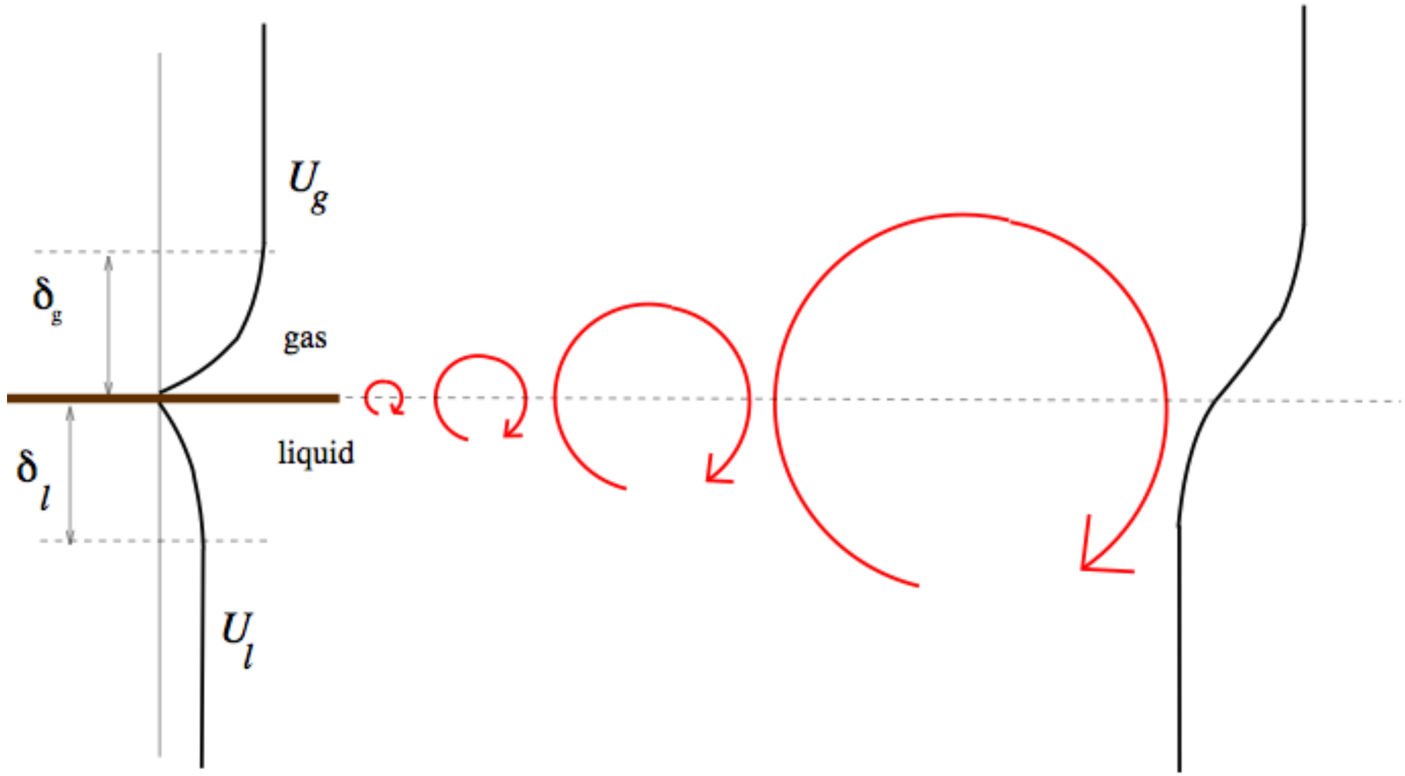


-Need for better parallelism. (although we get 6 billion grid points equivalent with 40 cores ...)

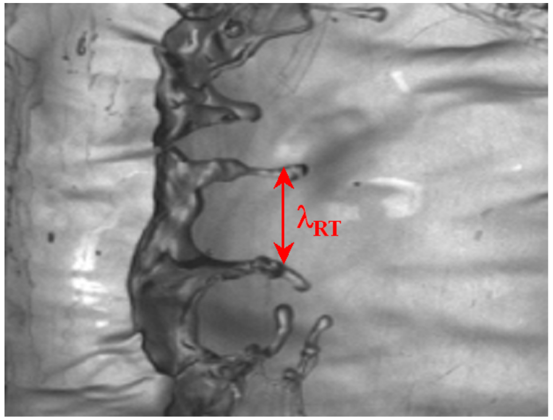
-Need for multiscale treatment : combine DNS with some type of subgrid modelling, for certain regions and certain physics, such as droplets in dilute regions (far from the core).

Interpretation 2: physical analysis of the transverse wavelength.

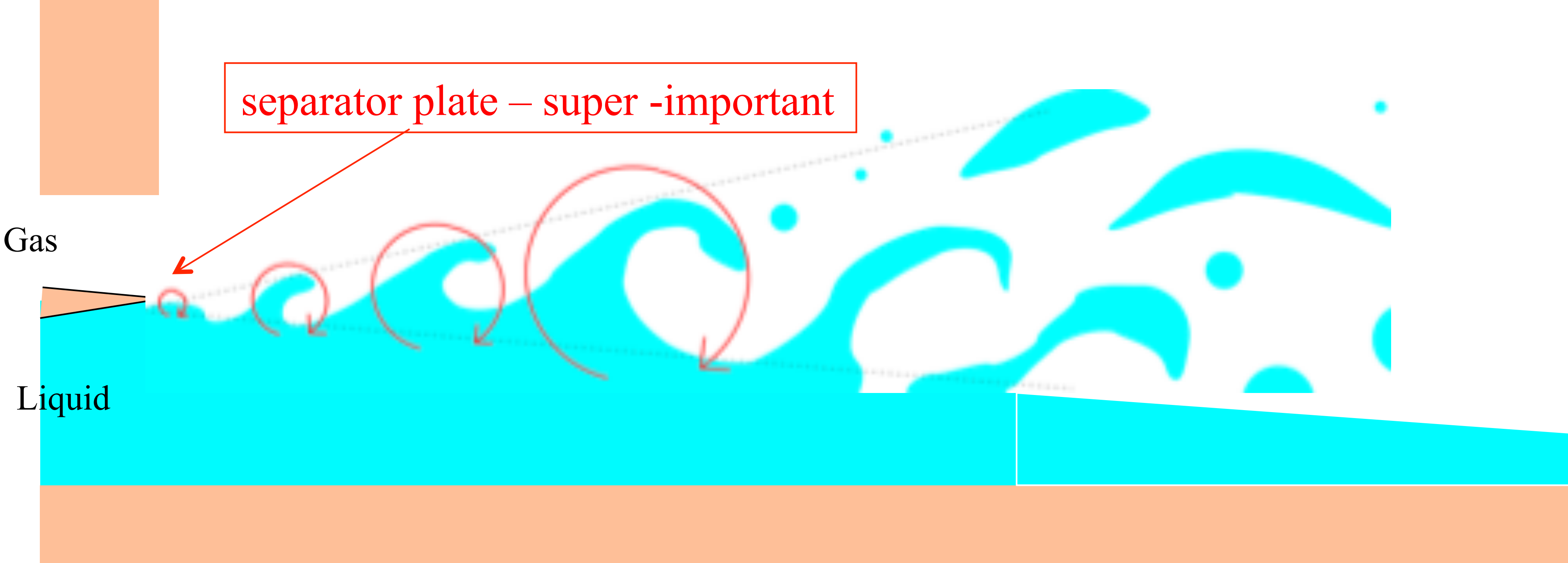
Analyze the flow as a spatially-developping mixing layer



Fully spatial 2D DNS to simulate setup of Grenoble's planar sheet experiment. Large-scale structures are 2D.




view from above

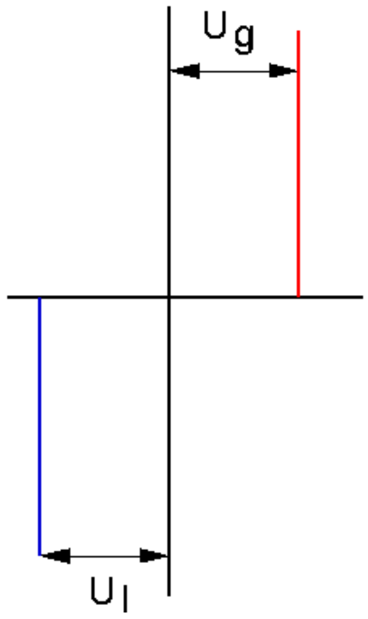


# A mechanism of Atomization : the Kelvin-Helmholtz Instability

Linear stability theory of the Kelvin-Helmholtz instability:

(a) Potential flow (except at interface), Inviscid, Piecewise linear profiles (Marmottant, Raynal, Villermaux, Cartellier, Matas, others ...)

(b) Viscous, Error-function profiles (Yecko, Fullana, Boeck, Zaleski, Gordillo, Perez-Saborid, Ganan-Calvo, Spelt, Valluri, O' Naraigh... )

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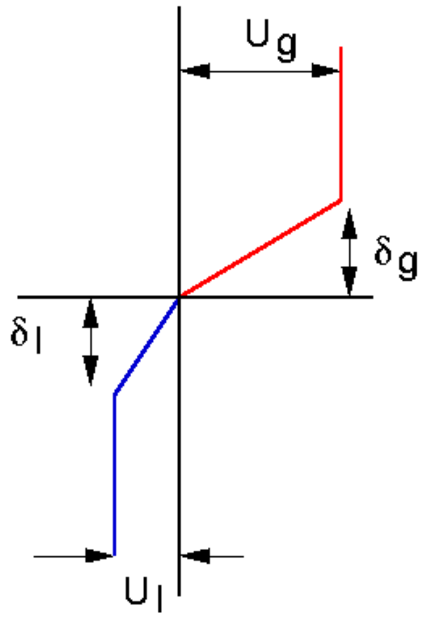


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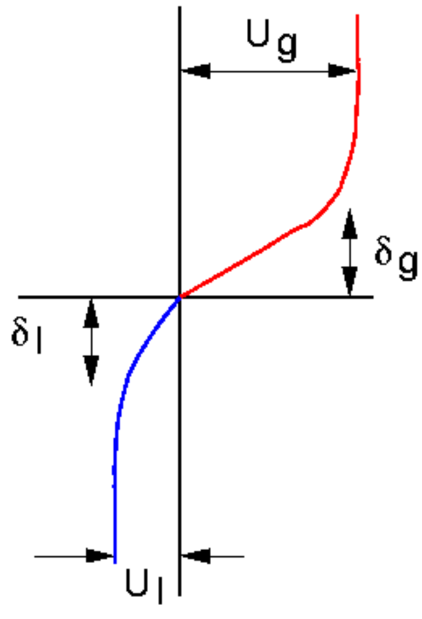



21 ...

# INVISCID LINEAR STABILITY PROBLEM

Nondimensional form based on gas layer velocity and thickness.

$$\Psi_{l,g} = \Phi_{l,g}(y)\exp(i\alpha(x - ct))\,,\; u = \partial_y \Psi\,,\; v = -\partial_x \Psi$$

$$\boxed{(U_{l,g} - c)(D^2 - \alpha^2)\Phi_{l,g} - D^2 U_{l,g}\Phi_{l,g} = 0}$$

boundary conditions on interface(s)

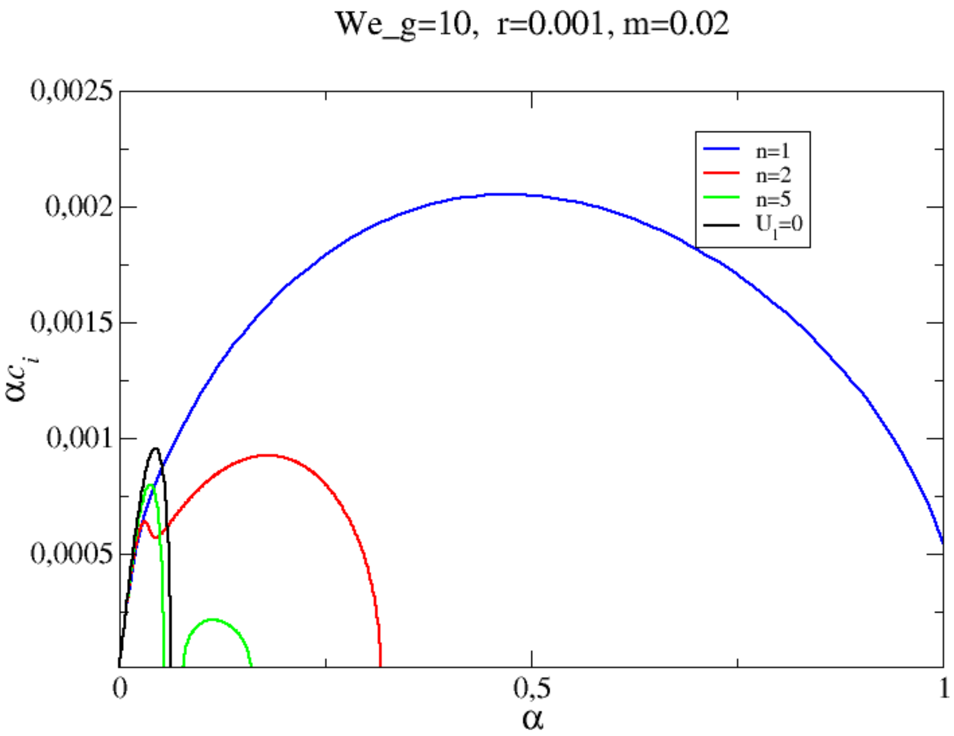
$\Phi_l = \Phi_g$ vertical velocity continuous

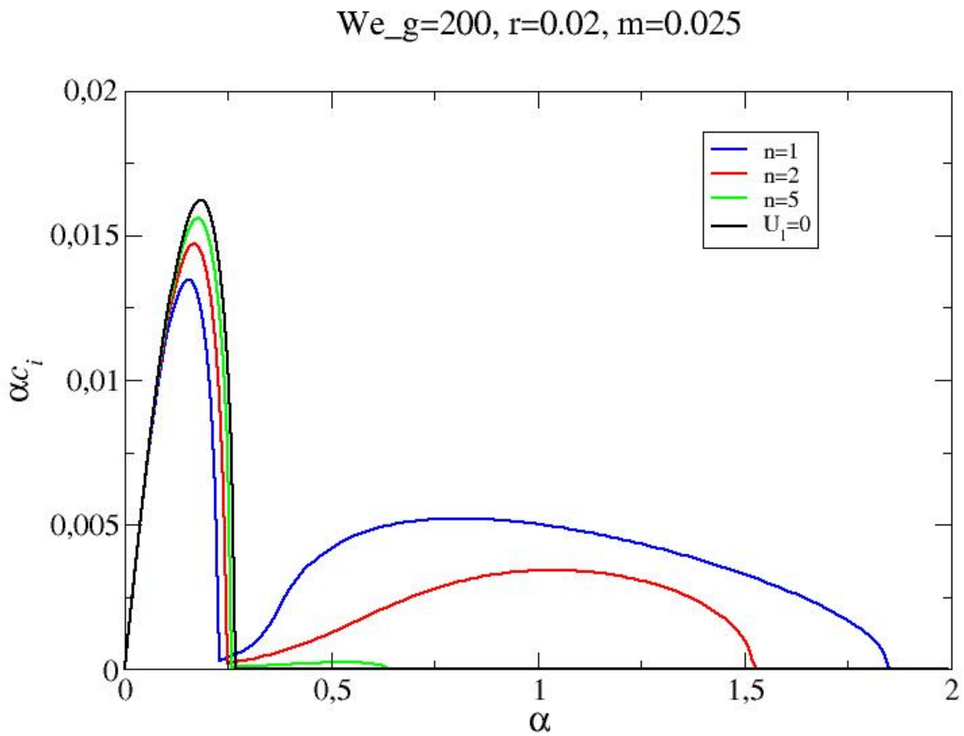
$$-\frac{\alpha^2}{cWe_g}\Phi_g = -\frac{1}{r}\left(cD\Phi_l + \Phi_l DU_l\right) + cD\Phi_g + \Phi_g DU_g$$ P cont.

Analytical solution when U linear function of y; dispersion relation then 4th order polynomial for the broken line profiles in liquid and gas phase.

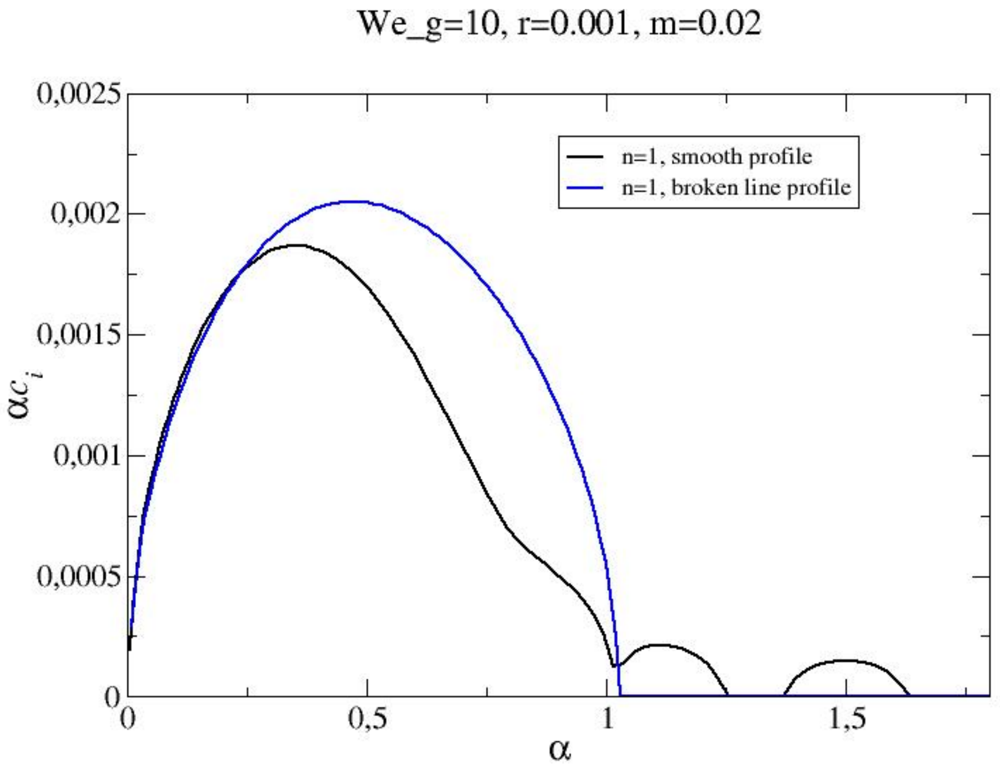
General profile: numerical solution with collocation method based on expansion in Chebyshev polynomials in both (finite) layers.

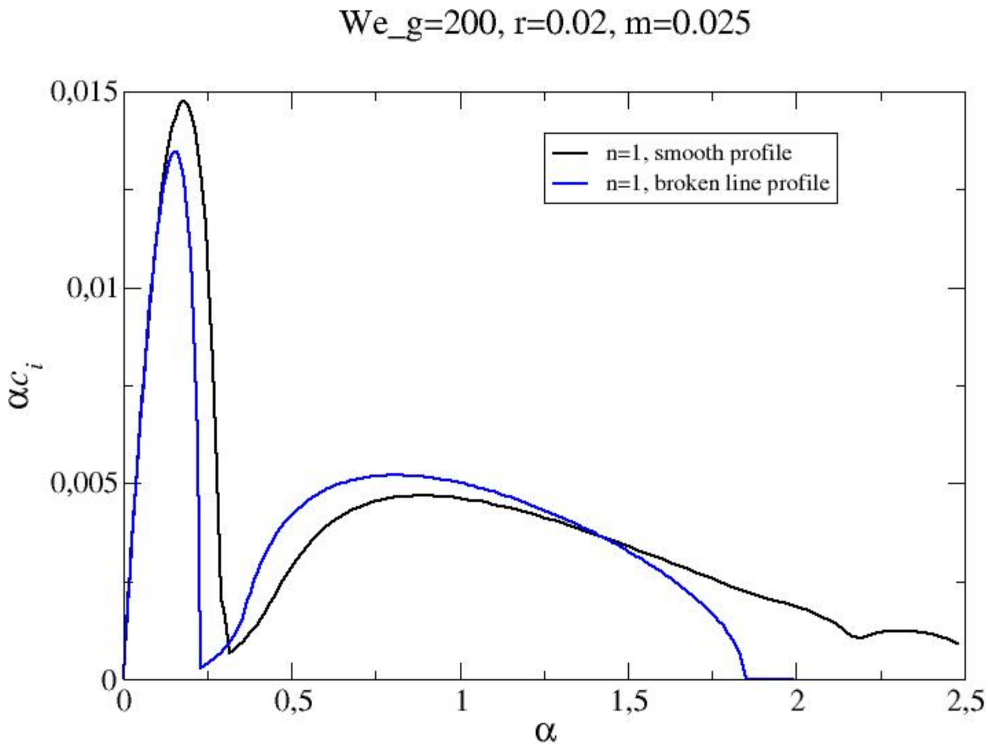
# Broken line profile



# Continuous profile



# Viscous linear stability problem

Nondimensional form based on gas layer velocity and thickness.

$$\Psi_{l,g} = \Phi_{l,g}(y)\exp(i\alpha(x-ct)) \,,\; u = \partial_y \Psi \,,\; v = -\partial_x \Psi$$

$$(U_g - c)(D^2 - \alpha^2)\Phi_g - D^2 U_g \Phi_g = \frac{1}{i\alpha \mathrm{Re}_g}(D^2 - \alpha^2)^2 \Phi_g$$

$$(U_l - c)(D^2 - \alpha^2)\Phi_l - D^2 U_l \Phi_l = \frac{r}{m}\frac{1}{i\alpha \mathrm{Re}_g}(D^2 - \alpha^2)^2 \Phi_l$$

<u>boundary conditions on interface(s)</u>

$$\Phi_l = \Phi_g$$ vertical velocity continuous

$$D\Phi_l + DU_l \frac{\Phi_l}{c} = D\Phi_g + DU_g \frac{\Phi_g}{c}$$ horizontal velocity continuous

# Viscous linear stability problem II

boundary conditions on interface(s)

$$m(D^2+\alpha^2+\frac{1}{c}D^2U_g)\Phi_g=(D^2+\alpha^2+\frac{1}{c}D^2U_1)\Phi_1$$

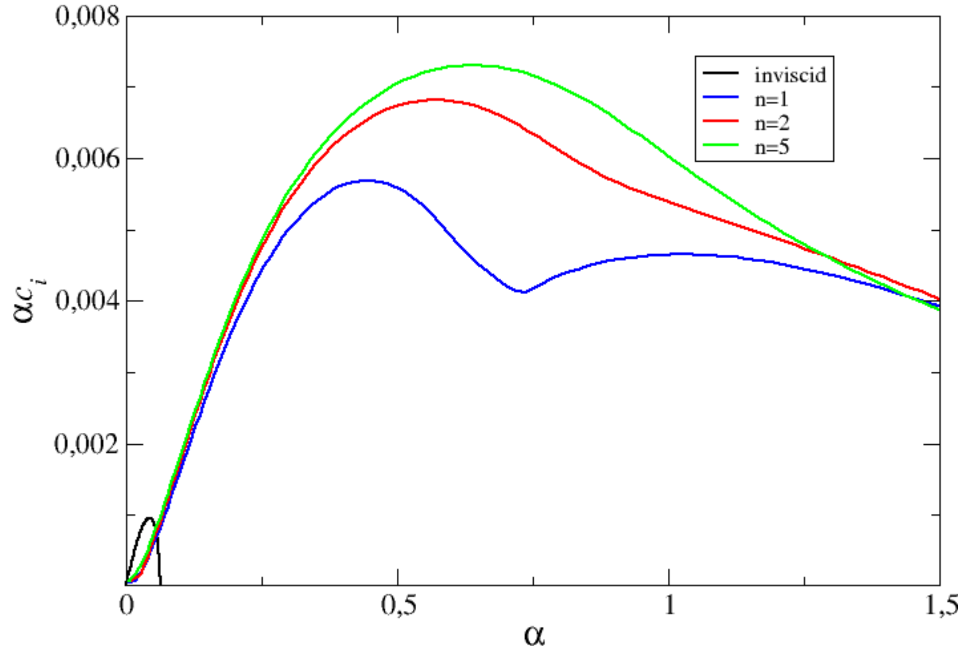
continuity of tangential stress

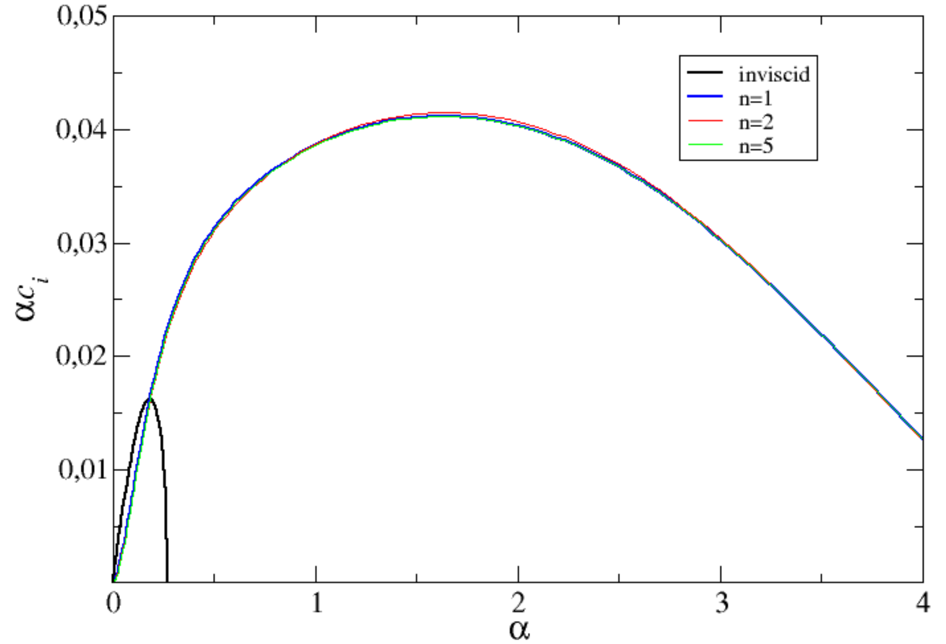
$$-\frac{\alpha^2}{cWe_g}\Phi_g=-\frac{1}{r}\left(cD\Phi_1+\Phi_1DU_1\right)-\frac{r}{m}\frac{1}{i\alpha Re_g}(D^3-3\alpha^2D)\Phi_1$$

$$+cD\Phi_g+\Phi_gDU_g+\frac{1}{i\alpha Re_g}(D^3-3\alpha^2D)\Phi_g$$

continuity of normal stress

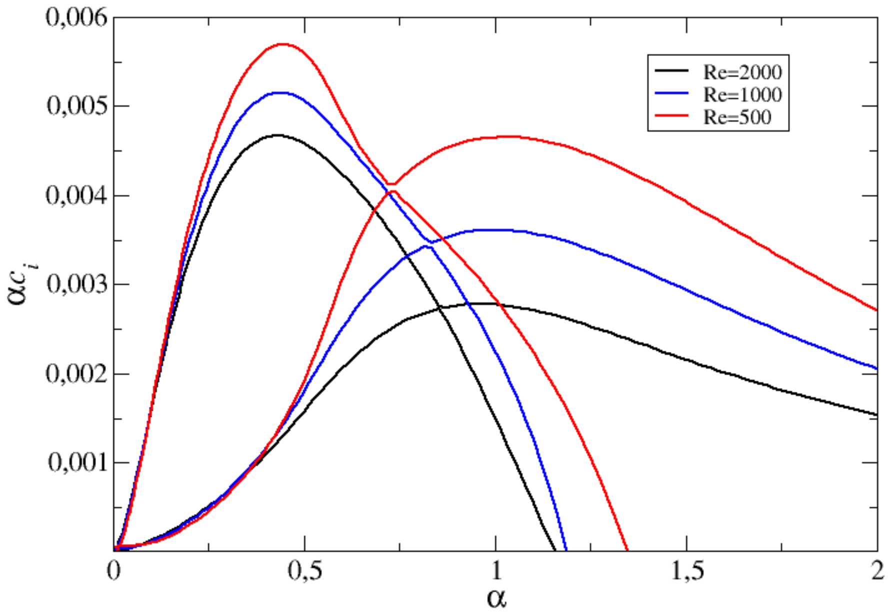
# viscous case



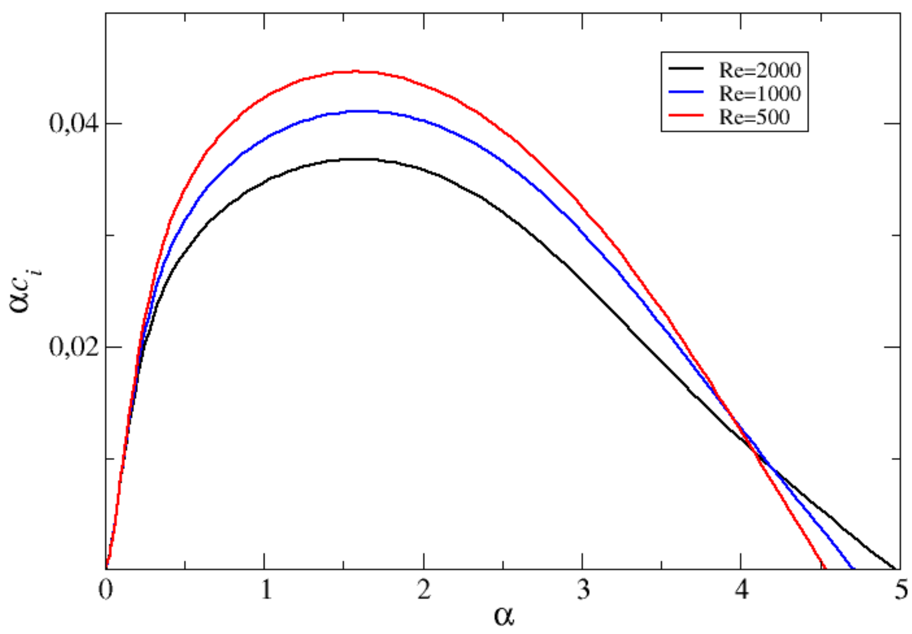



# Reynolds number influence

We_g=10, n=1, r=0.001, m=0.02




We_g=200, n=1, r=0.02, m=0.025

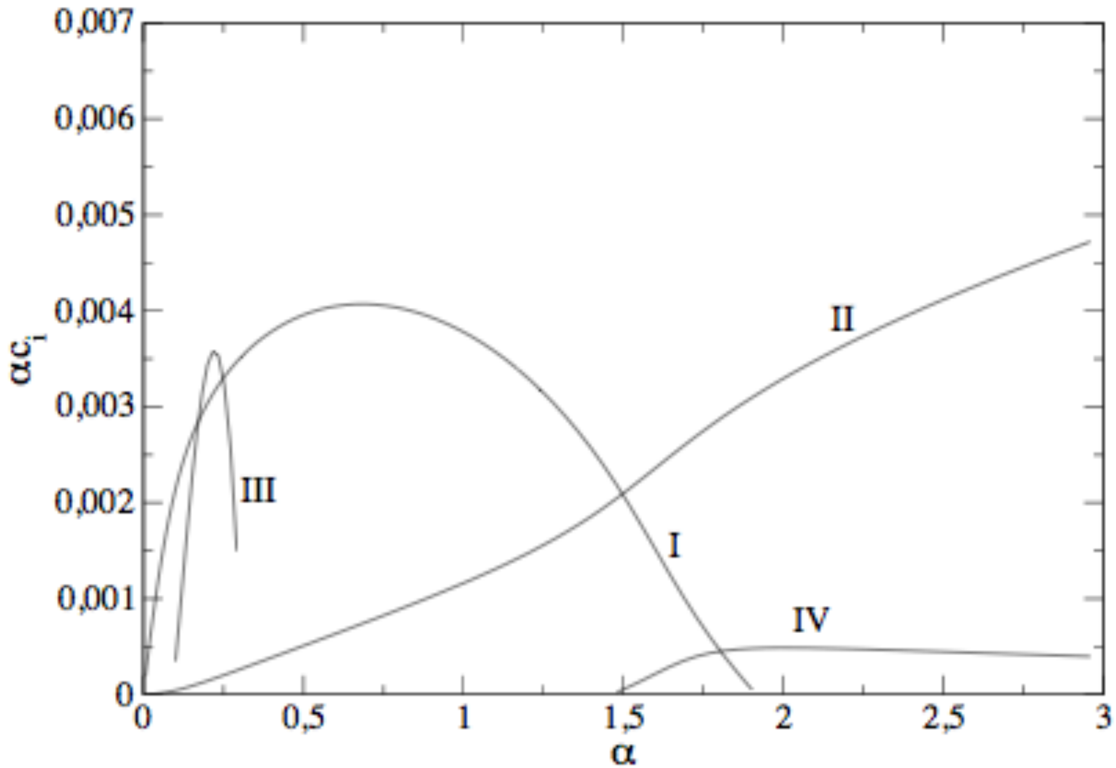


## Air-water case Re = $3 \times 10^4$ We = infinity

I : connects to inviscid KH at very high Re

II : H-mode (zero Re mode)

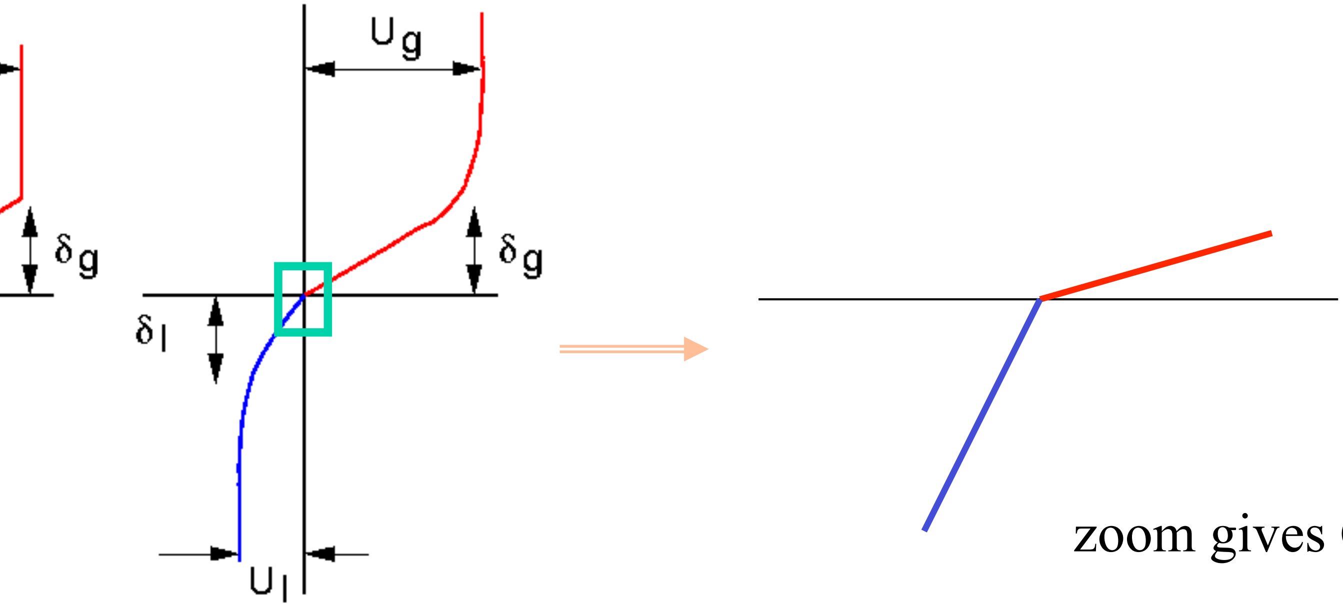
III : similar to Tollmien-Schlichting




It is the interaction of the H mode (mode I) and the inviscid mode (mode II) that explains the difference between the Orr-Sommerfeld and inviscid results.

# What is the new H mode ?

- named after Hooper and Boyd (1982) and Hinch (1984).
- a mode at Re=0
- zoom in on vicinity of the interface




zoom gives Couette flow

- The H mode is an instability of Couette flow at zero Re. The scaling of the H mode can be explained as follows:

- the only time scale in the shear layer is

$$1/\omega_g = \delta_g / U_g$$

thus the only length scale is

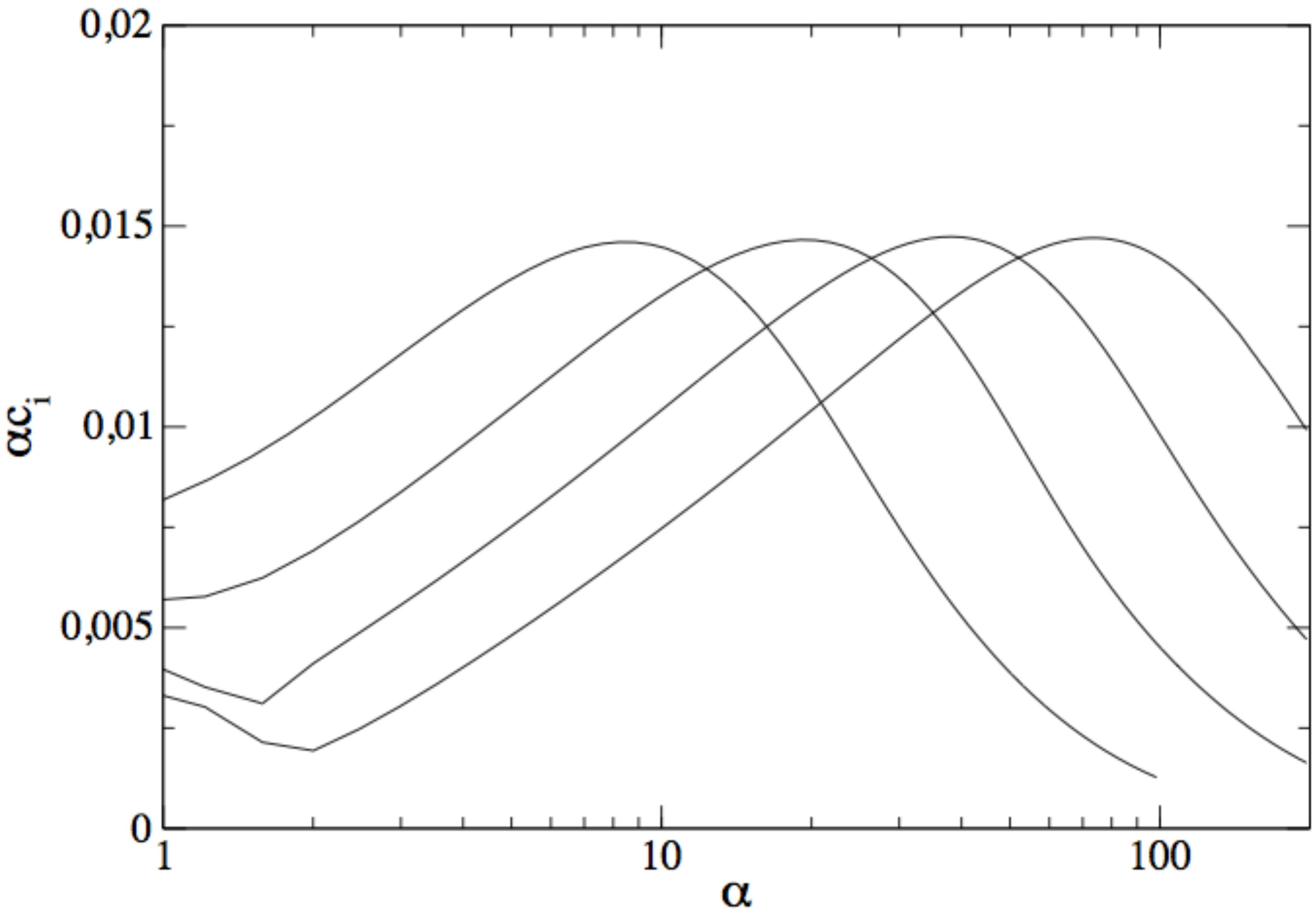
$$l = \sqrt{\frac{\nu_g}{\omega_g}} = \frac{\delta_g}{\sqrt{\mathrm{Re}_g}}$$

implies that the (dimensionless) most unstable wavenumber grows like

$$\sqrt{\mathrm{Re}_g}$$

H-mode branch is shifted in log diagram when Re is increased

Air-water case Re = 2 $10^3$ , $10^4$ , 4 $10^4$ , 1.2 $10^5$ , We = infinity



Numerical solutions of the Orr-Sommerfeld equations in the range of the experiments show that dimensional wavelength decreases like

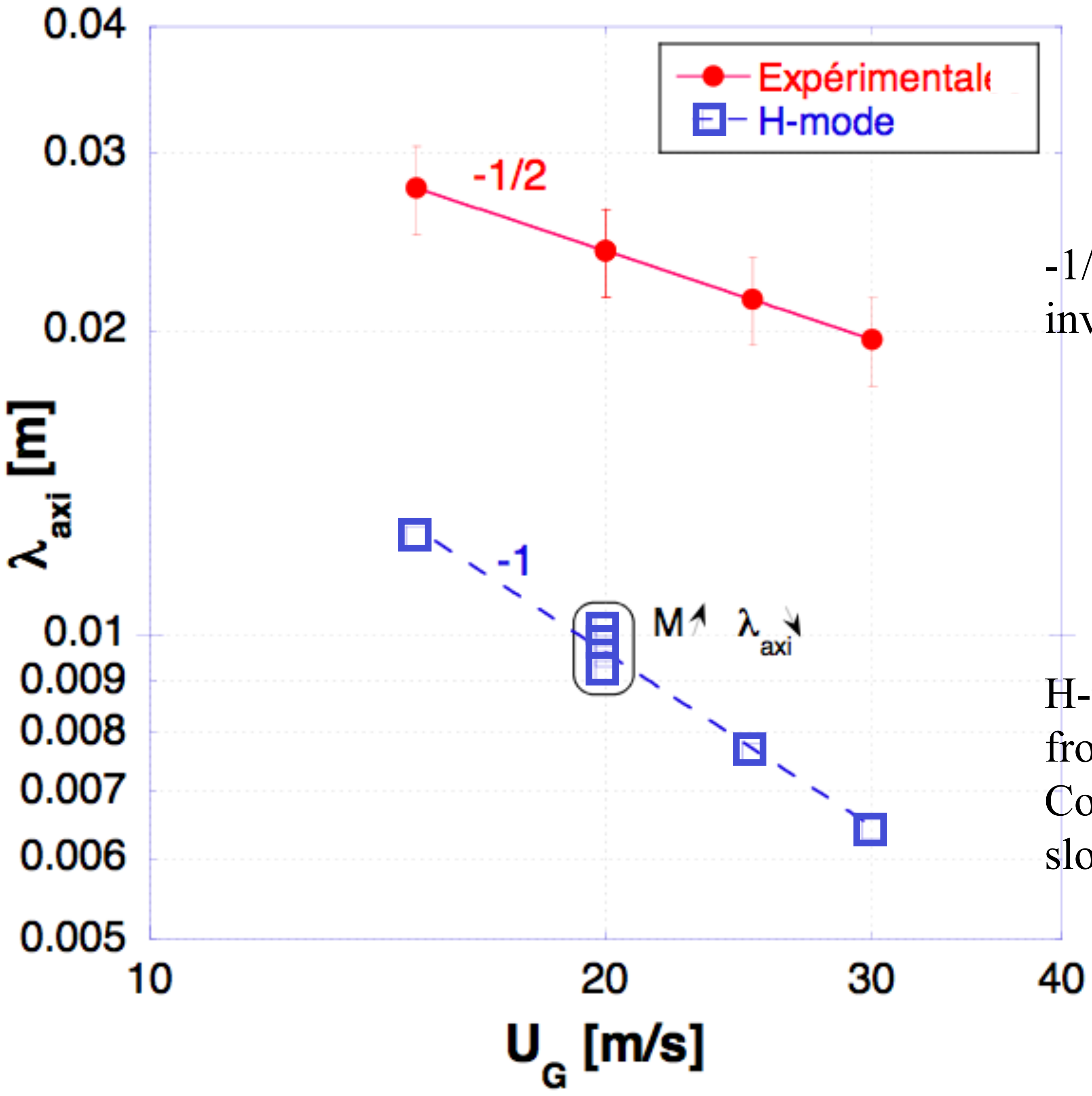
$$\lambda \approx U_g^{-1}$$

But experimental results show rather

$$\lambda \approx U_g^{-1/2}$$

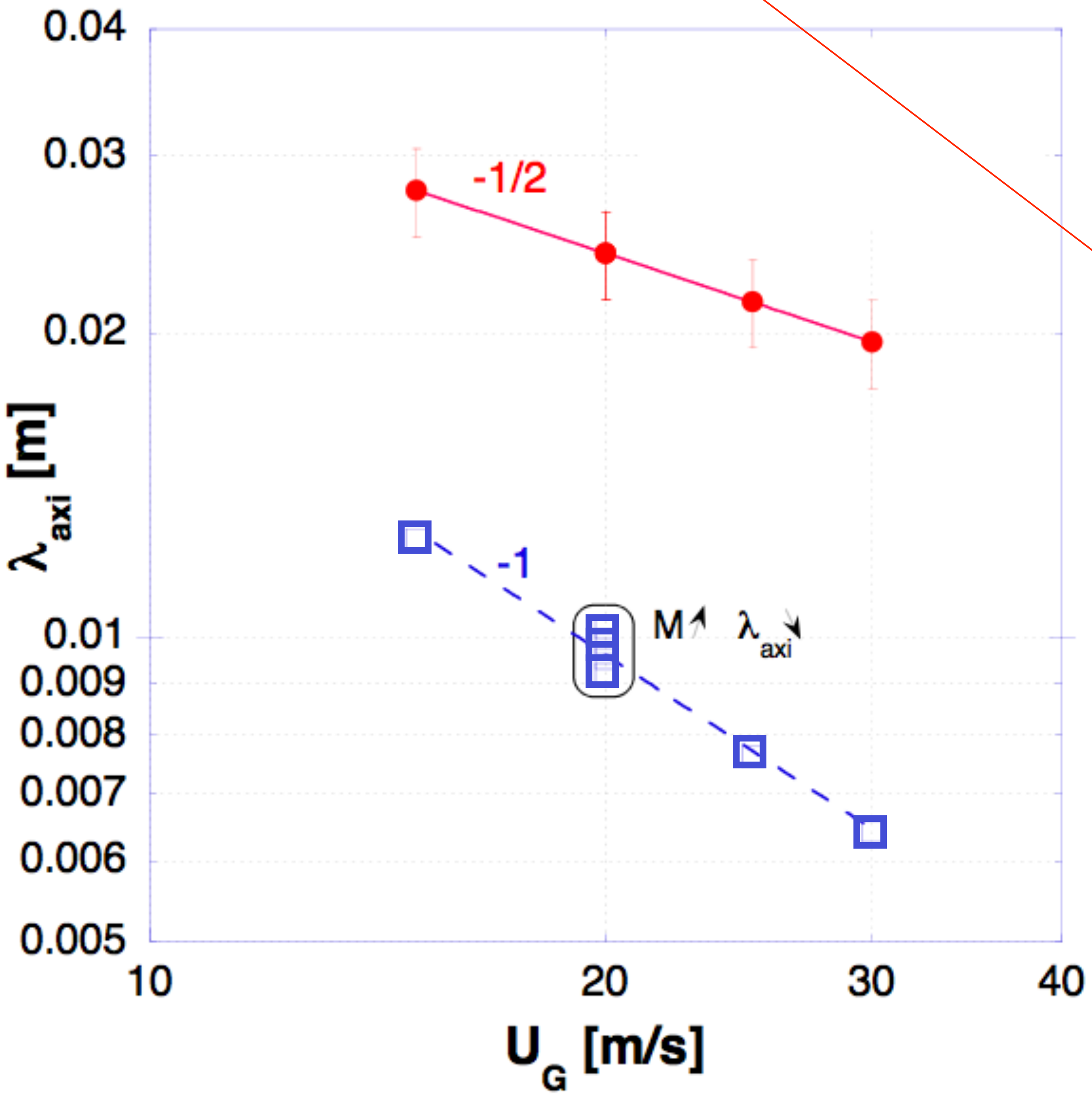
which is the result expected for the inviscid mode !

But the theory fails….




-1/2 slope from inviscid theory.

H-mode obtained from Orr-Sommerfeld Computations. -1 slope ! ! !




Inviscid theory
also « fails ».
Its prediction
are here ..
but with the
correct scaling.

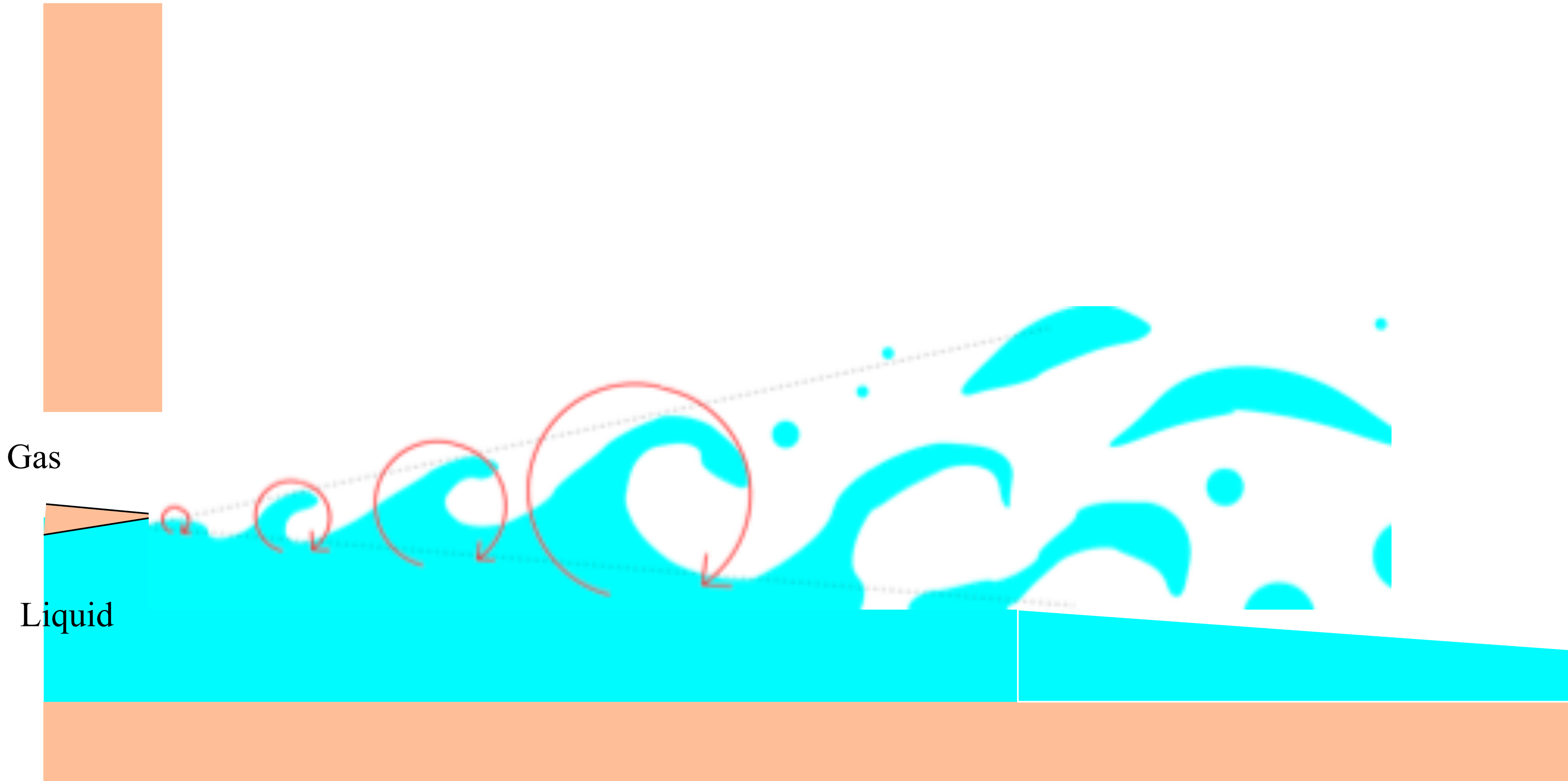
We aere convinced that the real world is viscous ... but the experiment can be interpreted only using inviscid scaling theories !

Simple explanations:

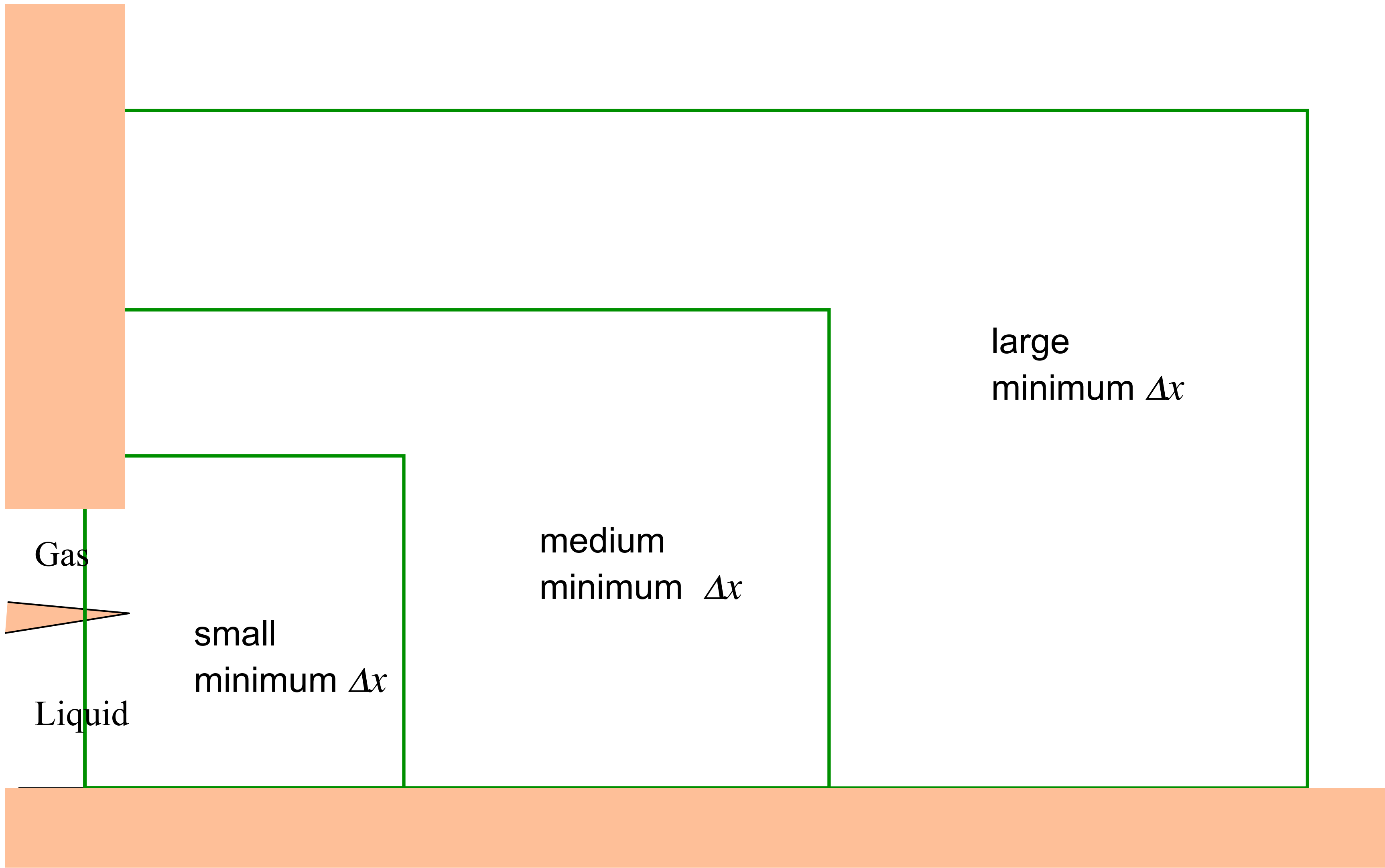
-the experiment is wrong (after all, it took « them » ten years to fix the errors in the initial measurements ☺ ).
-the Gaster transformation does not apply (but would this change the scaling?)

Gerris will at least allow us to verify the experiment

Back to the « Grenoble » 2D setup.



Elementary multiscale treatment: Navier-Stokes with variable minimum grid size according to a subdivision of the computational domain.

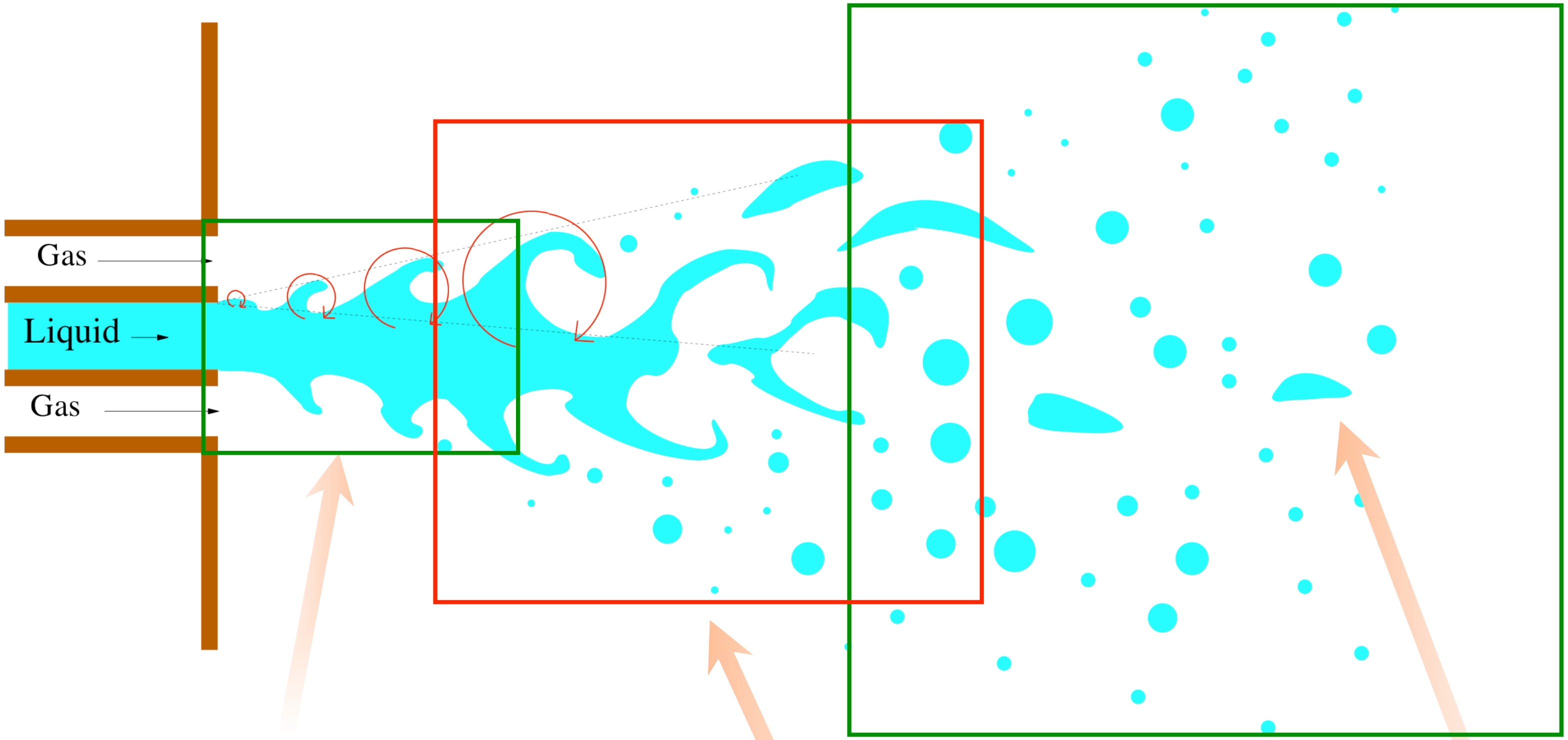


## More refined multiscale modelling ?

E and Enquist (Comm. Math. Sci, 2003) introduce the following typology :

- Type A problems : problems that require a localised effort to capture explicitly the smallest scales. Typically a boundary layer.

- Type B problems: large regions containing a homogeneous distributions of smaller sales for which an effective larger-scale model must be found. Typically the derivation of Navier-Stokes from kinetic theory, or an averaged multiphase-flow model.

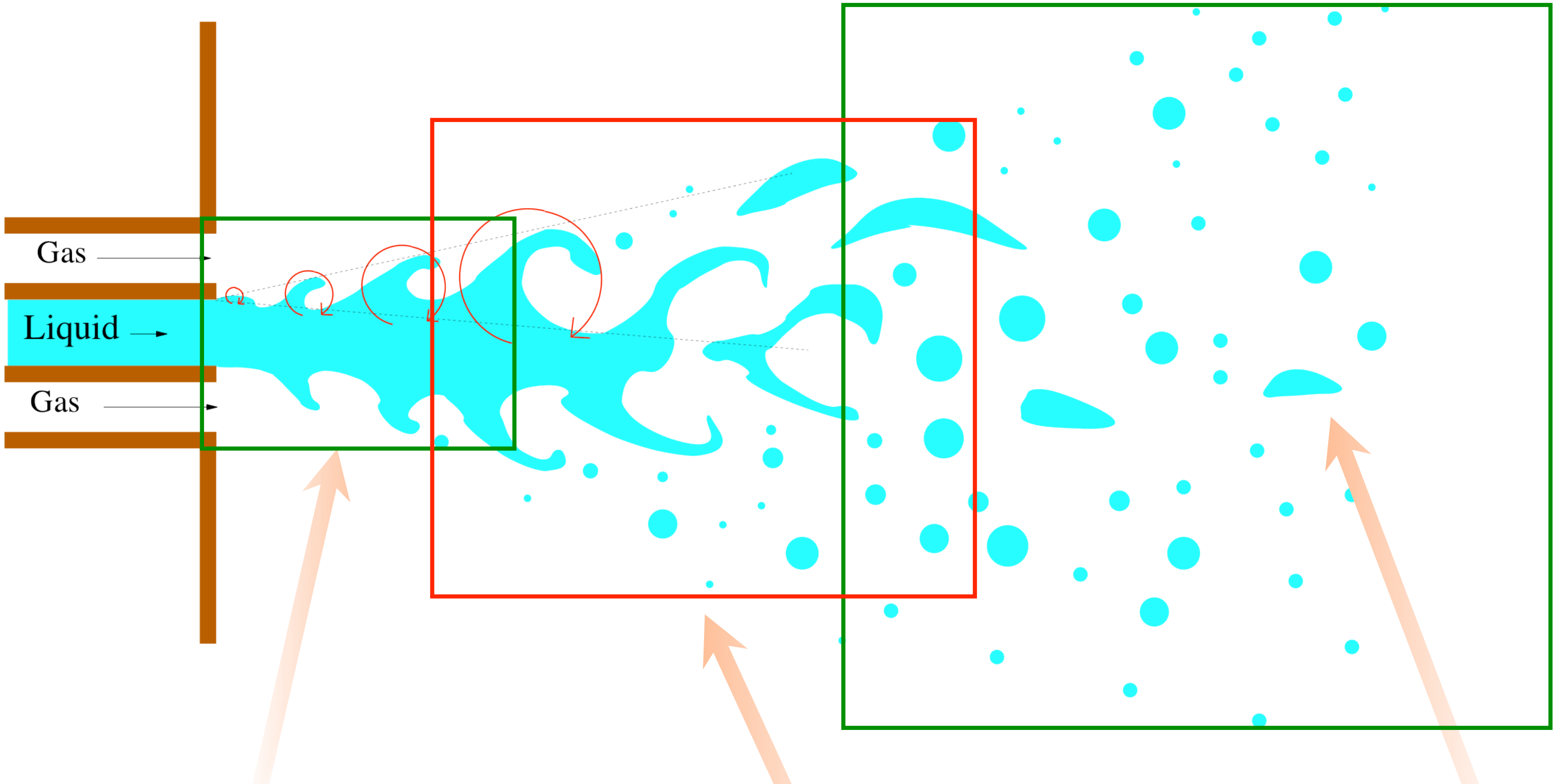



Dense spray –
convoluted interfaces :
Type A region.

Intermediate region: Type A
regions with progressive
coarsening downstream.

Type B region:
dilute spray.

# Simulation methodology

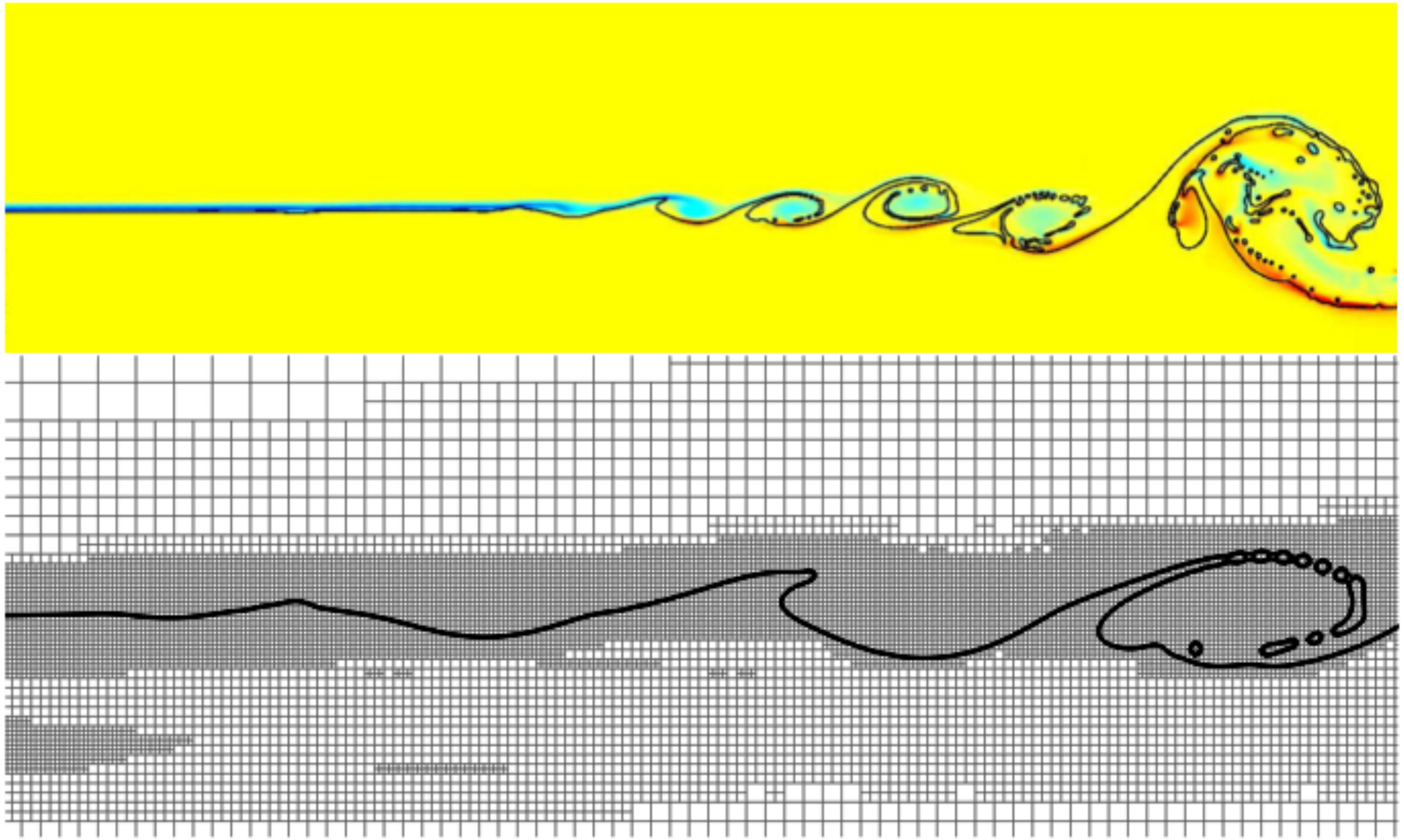



Dense spray – convoluted interfaces : need DNS. No subgrid scale model will work.

Intermediate region: requires work and subtelty.

Dilute spray: droplets may be accurately modelled as Lagrangian particles.

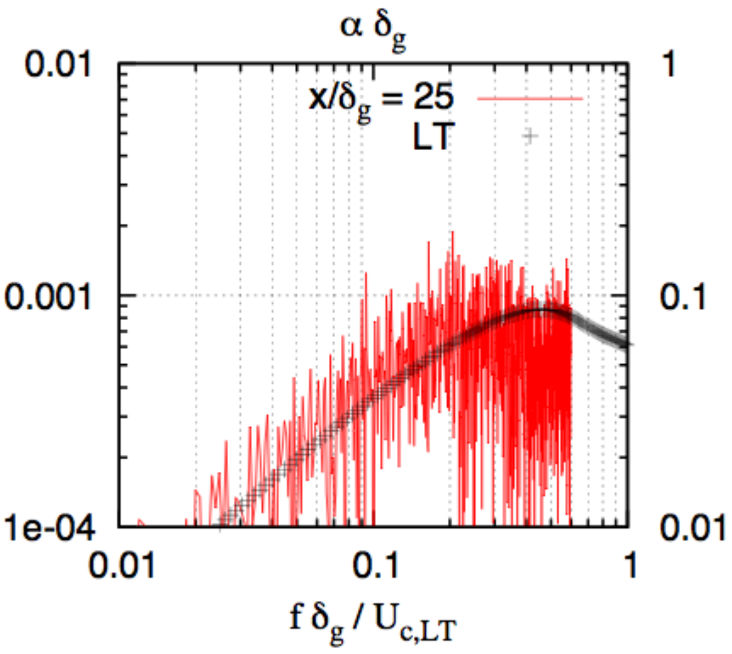
# Turn to full, spatially developping DNS



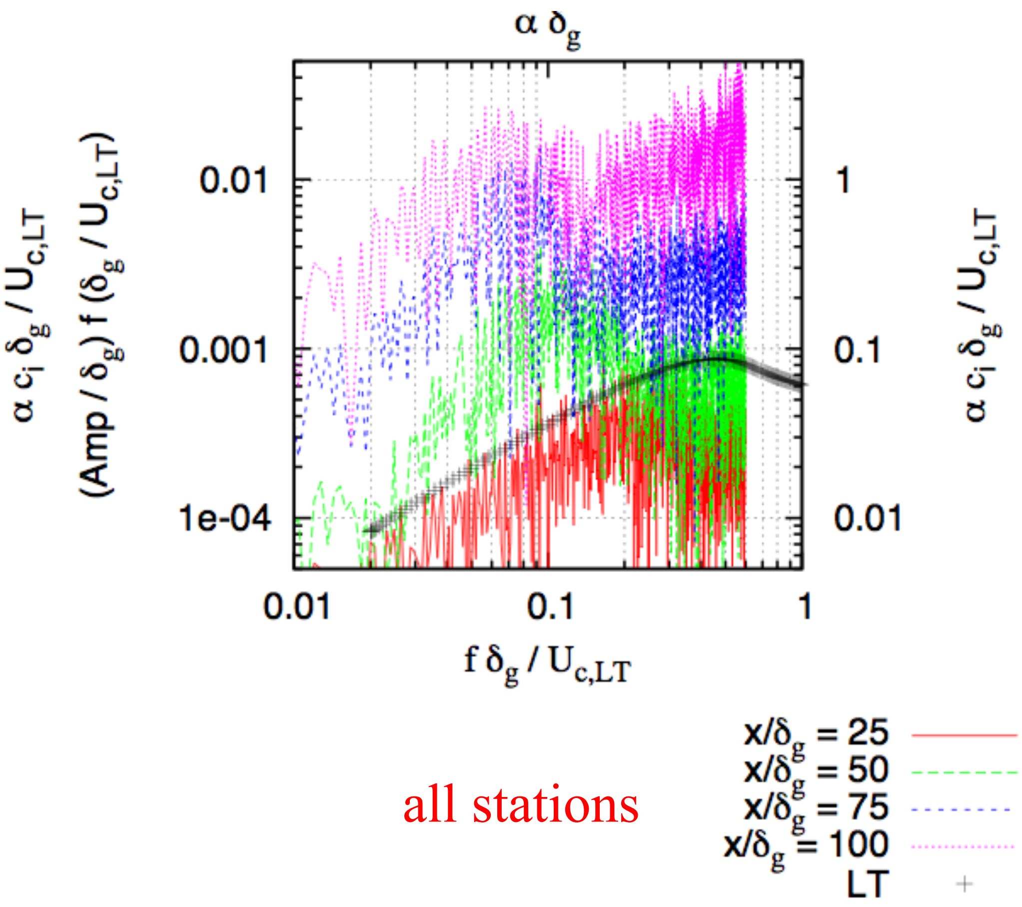
| $m$ | $r$ | $Re_l^*$ | $Re_g^*$ | $We_l^*$ | $We_g^*$ | $M$ |
|---|---|---|---|---|---|---|
| 0.034 | 0.200 | 471 | 2350 | 157 | 133 | 5.00 |

Fuster et al. 2009. IJMF

Viscous, Orr-Sommerfeld theory is verified !!




near nozzle

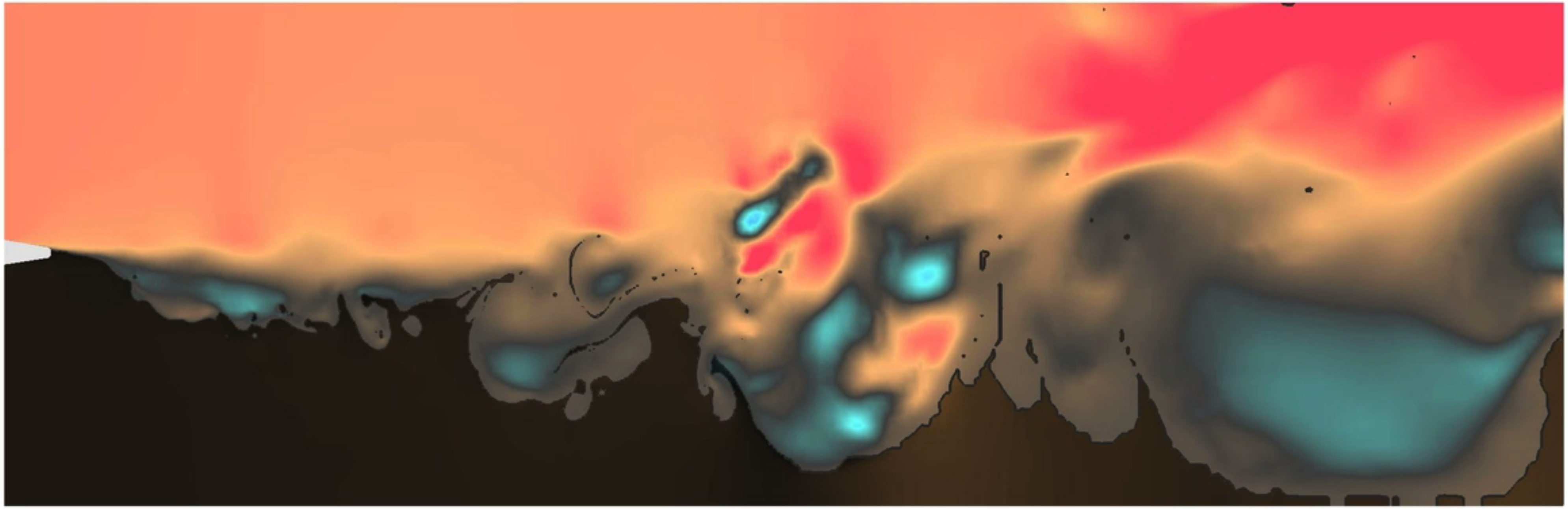



all stations

Not the real experimental parameters however (not air-water properties)

Simulation with a separator plate at larger density ratio (1/*r*)

| $m$ | $r$ | $Re_g$ | $Re_l$ | $We_g$ | $We_l$ | $M$ |
|---|---|---|---|---|---|---|
| 0.017 | 0,01 | 2640 | 290 | 19 | 8 | 2,4 |



Conclusion

We are not solving the problems that we think we are solving

# The End