Implementation of a symmetry-preserving discretization in Gerris

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Introduction

10/11: Recruited by the CNRS for a CR2 position

Topics of interest

- Multiphase flows
- Turbulent flows

Particular problems

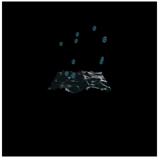
- Cavitation
- Atomization

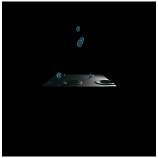
The basic idea is to develop numerical tools and models that allow us to investigate complex phenomena where multiple phases are involved.

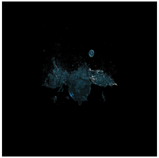
Introduction

Some code developed in Gerris

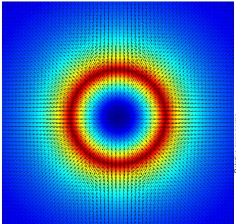
- Development of numerical schemes for the simulation of multiphase turbulent flows (col: Pierre Sagaut)
- Multi-scale phenomena
  - Development of numerical tools to simulate atomization processes (G. Tomar)

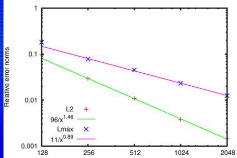






  - Investigation of cavitation processes: *Implementation of volumetric sources in Gerris*

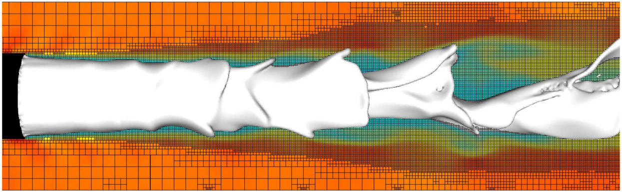




Multiphase turbulent flows

Idea: To take advantage of some Gerris' capabilities for the simulation of multiphase flows

- It is a code well validated for the DNS of two-phase flows
- It is flexible (complex geometries, different BC, etc...)
- It is parallelized
- Adaptive Mesh Refinement



Multiphase turbulent flows

The simulation of turbulent multiphase flows is complex

- Except for very simplified problems, DNS simulations are exceedingly expensive
- Small scale structures influence the dynamics of the interfaces
- The number of phenomena taking place in the sub-grid scale increase (small bubbles, droplets, mass and transfer heat)
- *The numerical methods developed for each of the flows have different characteristics*

Multiphase turbulent flows

Numerical methods applied to multiphase flows

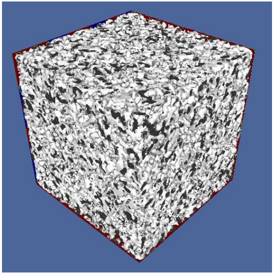
- **Desirable features:**
  - Designed to capture shocks
  - Usually stable
- **Negative:**
  Dissipative (unknown numerical dissipation)

Numerical methods applied to turbulent flows

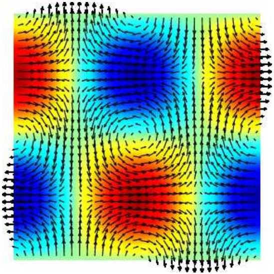
- **Desirable features:** Energy conservation is important
- **Negative:** These schemes are not always stable and may be less accurate

We can evaluate the performance of the code for two different scenarios:

- Isotropic turbulence



- Taylor-Green Vortex



Isotropic turbulence: We compare the solution of the code with an spectral code. at different Reynolds number (hit3d, Sergei Chumakov, Standford)

1. We generate random velocity field in Fourier space
2. We project the random field to obtain a solenoidal field
3. We rescale the spectra according to

$$E(k) = \alpha\epsilon^{2/3} k^{-5/3} f_L(kL) f_\nu(kLRe_L^{-3/4})$$

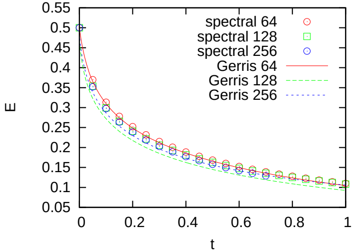
which is calculated imposing the value of the integral length

$$L_{\mathrm{int}} = 0.5L_{\mathrm{box}} = \pi,$$

and also the level of turbulent kinetic energy to 0.5,

$$0.5 = \int_0^\infty E(k)dk.$$

Isotropic turbulence



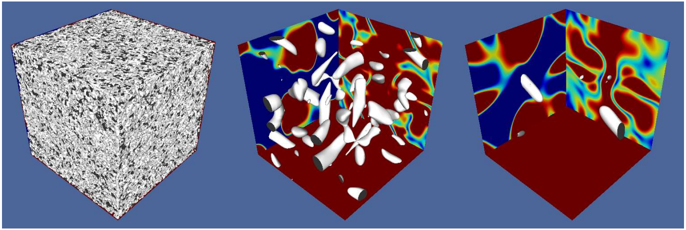
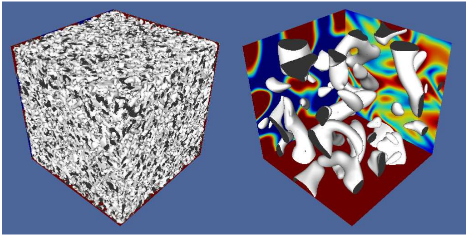
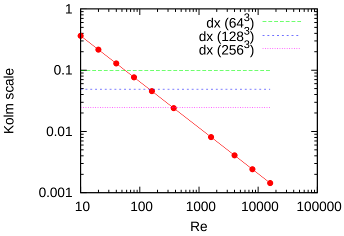


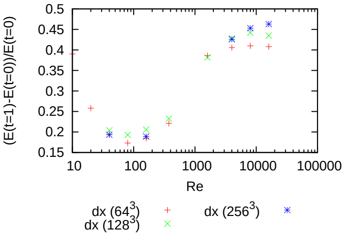
Figure: Temporal evolution of the energy decay at $Re_L = 100$ for different mesh resolutions and snapshots of the solution at t=0;0.5,1.

Isotropic turbulence



The Godunov scheme introduces numerical dissipation





Can we do better?

- In the inviscid case, the only source of numerical dissipation is the discretization of the non-linear convective term
- Verstappen et al (JCP, 2007) has proposed a conservative scheme that preserved energy at expenses of increases the local truncation error

Incompressible Navier Stokes

$$\begin{aligned} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{Re} \nabla \cdot \nabla \mathbf{u} + \nabla p = 0 \\ \nabla \cdot \mathbf{u} = 0 \end{aligned}$$

As the following properties are satisfied

$$(\mathbf{u} \cdot \nabla)^* = -(\mathbf{u} \cdot \nabla) \quad \nabla^* = -\nabla$$

The total energy of the system evolves as

$$(\mathbf{u}, \mathbf{u})_t = -\frac{2}{Re} (\nabla \mathbf{u}, \nabla \mathbf{u})$$

The idea is to derive a discretization scheme that satisfies these properties.

Skew-Symmetric formulation

The discrete version of the Navier Stokes equations reads

$$\Omega_0 \frac{d\mathbf{u}_b}{dt} + \mathbf{C}_0(\mathbf{u}^f)\mathbf{u}_h + \mathbf{D}_0\mathbf{u}_h = 0$$

when $\mathbf{D}_0 = 0$, the energy $||\mathbf{u}_h||^2 = \mathbf{u}_h^*\Omega_0\mathbf{u}_h$ is preserved if

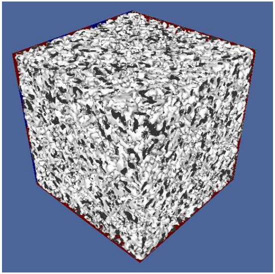
$$\frac{d}{dt}||\mathbf{u}_h||^2 = -\mathbf{u}_h^*(\mathbf{C}_0(\mathbf{u}^f) + \mathbf{C}_0^*(\mathbf{u}^f))\mathbf{u}_h = 0$$

Therefore the only conditions that must be satisfied is

$$\mathbf{C}_0(\mathbf{u}^f) + \mathbf{C}_0^*(\mathbf{u}^f) = 0$$

We finally obtain the following a discrete momentum equation
What about its performance?

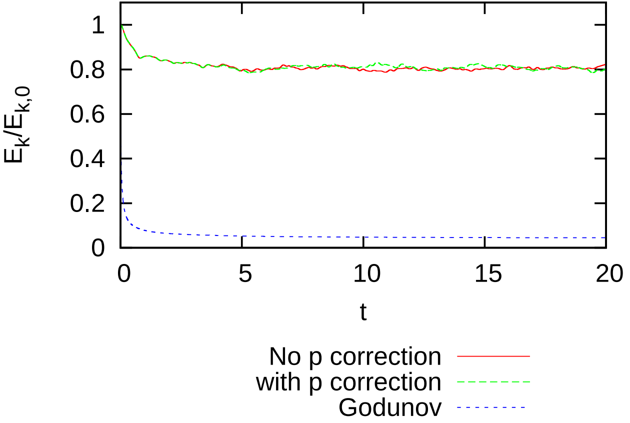
Isotropic turbulence test



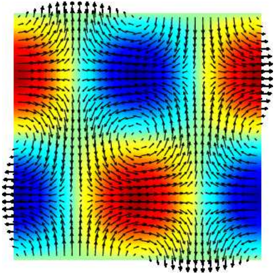
Skew-symmetric formulation

Isotropic turbulence

The method seems to behave relative well for infinite Reynolds number (Euler eqs.) where the results obtained using Godunov are very poor.

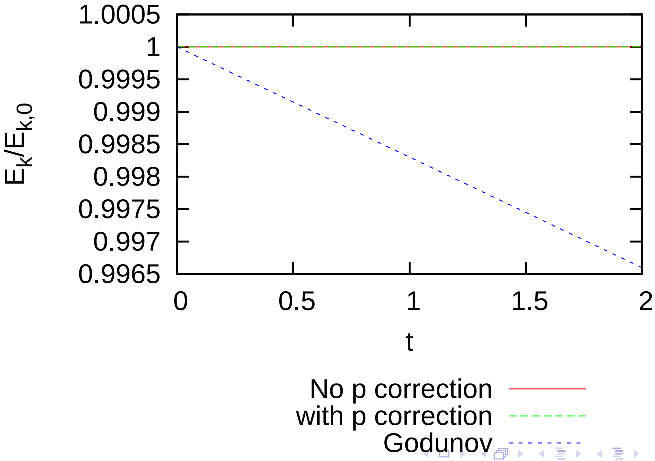


Green Taylor Vortex test



Green Taylor Vortex

$$U = -\cos(2\pi x)\sin(2\pi y)$$
$$V = \sin(2\pi x)\cos(2\pi y)$$



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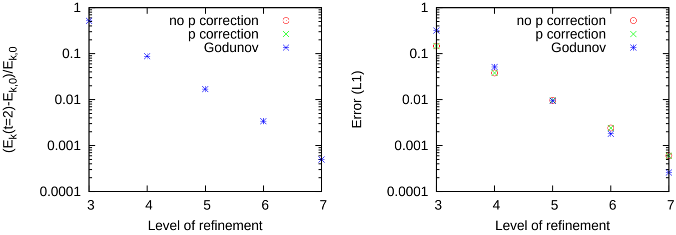

Figure: Energy conservation and accuracy of the different methods (skew-symmetric formulations preserve energy within machine accuracy).

Remarks

- Godunov displays better convergence
- Skew-Symmetric is more accurate at low spatial resolution

Work in progress...

I have tested that the current implementation of the skew-symmetric formulation allows us already to use some features of the code:

- Advect a VOF tracer
- 3D computations
- Paralelization

Pending to do:

- Adaptive Mesh Refinement: I am experiencing difficulties to extend the method to AMR
- Source terms: Especially care must be taken to introduce surface tension effects
- Boundary conditions: Because of the different nature of the numerical scheme, the extension to other types of Boundary Conditions is not straightforward

Conclusions

- Some projects have been started in the context of the simulation of multiphase and turbulent flows
- The development of numerical methods as well as multiscale models is oriented towards the development of computational tools for the simulation of complex processes.